# O(3) INVARIANCE OF THE AHARONOV BOHM EFFECT
by

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## ABSTRACT

It is shown that the Aharonov Bohm effect is not consistently described in the received view. A self-inconsistency is demonstrated in the U(1) gauge theory applied to electrodynamics that is the basis of the effect. A self-consistent description of the effect is suggested using a novel O(3) invariant form of electrodynamics, a form which also reproduces the experimental data available on the effect.


## 1 INTRODUCTION

The Aharonov Bohm effect[1]-[3] has been frequently discussed in the literature of the past forty years and has been the catalyst for several important developments[4]-[6]. In this paper it is shown that there is a self-inconsistency in the received theoretical view of the effect, which is based on U(1) gauge theory applied to electrodynamics[7]. The inconsistency is discussed in section 2. In section 3 a novel O(3) invariant electrodynamics[8]-[17] is applied in order to remove the self-inconsistency and to achieve agreement with the available experimental data on the effect. The result is conclusive evidence that the Aharonov Bohm effect is O(3) invariant, i.e. stems from an O(3) invariant lagrangian density in electrodynamics. The effect is therefore another piece of evidence in support of the thesis[8]-[17] that electrodynamics is in general an O(3) invariant gauge field thoery.

## 2 INCONSISTENCY IN THE U(1) INVARIANT GAUGE THEORY

In the U(1) invariant electrodynamics[7] which are usually applied to explain the Aharonov Bohm effect, it is convenient to illustrate the self-inconsistency with reference to the experiment where a solenoid is placed between the apertures of a Young interferometer, causing a shift in the interference pattern of matter waves, for example electron waves[7]. It is well known that the magnetic field $\mathbf{B}$ in this case is confined to the solenoid, but that the vector potential $\mathbf{A}$ is non zero outside the solenoid. In the U(1) invariant electrodynamics:

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{1}$$

The received view[7] of the Aharonov Bohm effect proceeds by noting that outside the solenoid:

$$\mathbf{B} = \mathbf{0}; \quad \nabla \times \mathbf{A} = \mathbf{0} \tag{2}$$

so that

$$\mathbf{A} := \nabla \chi \tag{3}$$

The observable phase shift difference for different electron wave paths is therefore given by:

$$\Delta\delta = \frac{e}{\hbar}\oint \nabla\chi \cdot d\mathbf{r} = \frac{e}{\hbar}\Big[\,\chi\,\Big]_0^{2\pi} = \frac{e}{\hbar}\int \mathbf{B} \cdot d\mathbf{S} \tag{4}$$

where $\chi$ is a periodic function. However, the received description[7] also uses the result:

$$\Delta\delta = \frac{e}{\hbar}\int \nabla \times \mathbf{A} \cdot d\mathbf{S} = \frac{e}{\hbar}\int \mathbf{B} \cdot d\mathbf{S} \tag{5}$$

This is clearly self-inconsistent from eqn.(2), which gives the result:

$$\Delta\delta = \frac{e}{\hbar}\int \nabla \times \mathbf{A} \cdot d\mathbf{S} = 0 \tag{6}$$

Barrett[17] has succinctly described this self-inconsistency as being due to the fact that there is no magnetic field present at the point of contact with the electron wave.

## 3 O(3) INVARIANT DESCRIPTION OF THE AHARONOV BOHM EFFECT

This description is based on the concept of phase factor[15]-[17] and the general theorem[18] that the phase factor is due to parallel transport for any internal gauge field symmetry such as U(1)[7] or O(3)[8]-[17]. The parallel transport occurs in a closed loop and gives the general theorem[14]:

$$\mathsf{g}\oint \mathsf{D}_\mu dx^\mu = -\frac{1}{2}\mathsf{g}\int [\mathsf{D}_\mu, \mathsf{D}_\nu] d\sigma^{\mu\nu} \tag{7}$$

where g is a topological charge and where $\mathsf{D}_\mu$ is a covariant derivative. In a U(1) invariant electrodynamics eqn.(7) becomes the Stokes Theorem:

$$\oint \mathbf{A} \cdot d\mathbf{r} = \int \nabla \times \mathbf{A} \cdot d\mathbf{S} = \int \mathbf{B} \cdot d\mathbf{S} \tag{8}$$

which is the basis for the U(1) invariant description of the Aharonov Bohm effect, i.e. eqn.(4).

Recently it has been demonstrated by several authors[8]-[17] that electrodynamics can be O(3) invariant, and that such an electrodynamics has several advantages over the received U(1) invariant version. In the O(3) invariant version there exists an internal gauge space of O(3) symmetry, a space which can be described by the complex basis $((1),(2),(3))$[11]-[14]. There occurs[14] an additional O(3) invariant Stokes Theorem:

$$\oint A_Z^{(3)} dZ = \int B_Z^{(3)} dAr \tag{9}$$

and O(3) invariant phase factor:

$$\mathsf{exp}\Big(ig\oint A_Z^{(3)}dZ\Big) \quad = \quad \mathsf{exp}\Big(ig\int B_Z^{(3)}dAr\Big) \tag{10}$$

If $\mathbf{B}^{(3)}$ of the O(3) invariant gauge theory is identified with the magnetic field of the solenoid, then the change in phase difference equivalent to eqn.(4) of the U(1) invariant gauge theory is given in the O(3) invariant gauge theory by;

$$\Delta\delta \quad = \quad \frac{e}{\hbar}\oint \mathbf{A}^{(3)}\cdot d\mathbf{r} \quad = \quad \frac{e}{\hbar}\int \mathbf{B}^{(3)}\cdot d\mathbf{S} \tag{11}$$

An O(3) gauge transformation produces[8]-[17]:

$$\mathbf{A}^{(3)} \rightarrow \mathbf{A}^{(3)} + \frac{1}{g}\frac{\partial\alpha}{\partial Z}\mathbf{e}^{(3)} \tag{12}$$

and

$$\mathbf{B}^{(3)} \rightarrow \mathsf{S}\mathbf{B}^{(3)}\mathsf{S}^{-1} = \mathbf{B}^{(3)} \tag{13}$$

where $\mathsf{S}$ is an exponential operator defined by:

$$\mathsf{S} = \mathsf{exp}\Big(i\mathsf{M}^a\Lambda^a(x^\mu)\Big) \tag{14}$$

where $\Lambda^a$ are angles and $\mathsf{M}^a$ are generators of the O(3) group. In areas outside the solenoid therefore the O(3) invariant Aharonov Bohm effect is given by the observed[7]:

$$\Delta\delta \quad = \quad \frac{e}{\hbar}\int \mathbf{B}^{(3)}\cdot d\mathbf{S} \quad := \quad \frac{e}{\hbar}\Phi \tag{15a}$$

$$\Delta\delta \quad = \quad \frac{e}{\hbar}\,\frac{1}{\mathsf{g}}\oint \frac{\partial\alpha}{\partial Z}\,\mathbf{e}^{(3)}\cdot d\mathbf{r} \tag{15b}$$

and there is a magnetic field present at the point of contact with the matter wave as required[17]. The term on the right hand side of eqn.(15b) is a physical term because the generator $\mathsf{S}$ consists of physical angles in a physical internal space of O(3) symmetry in the basis $((1),(2),(3))$[14]. Therefore an O(3) invariant theory gives a straightforward explanation of the Aharonov Bohm effect whereas the U(1) invariant explanation is self contradictory.

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