COMMENT ON THE LORENTZ INVARIANCE OF THE B CYCLIC THEOREM

by

Petar K. Anastasovski[1], T. E. Bearden[2], C. Ciubotariu[3], W. T. Coffey[4], L. B. Crowell[5], G. J. Evans[6], M. W. Evans[7,8], R. Flower[9], A. Labounsky[10], B. Lehnert[11], M. Mészáros[12], P. R. Molnár[12], J. K. Moscick[13], S. Roy[14], and J. P. Vigier[15]

Institute for Advanced Study, Alpha Foundation, Institute of Physics,
11 Rutafa Street, Building H, Budapest H-1165, Hungary

Also at:

[1]Faculty of Technology and Metallurgy, Department of Physics,
University of Skopje, Republic of Macedonia
[2]CEO, CTEC Inc., 2311 Big Cove Road, Huntsville, AL 35801-1351.
[3]Institute for Information Technology, Stuttgart University, Stuttgart, Germany
[4]Department of Microelectronics and Electrical Engineering,
Trinity College, Dublin 2, Ireland.
[5]Department of Physics and Astronomy, University of New Mexico,
Albuquerque, New Mexico
[6]Ceredigion County Council, Aberaeron, Wales, Great Britain
[7]former Edward Davies Chemical Laboratories, University College of Wales,
Aberystwyth SY23 1NE, Wales, Great Britain
[8]sometime JRF, Wolfson College, Oxford, Great Britain
[9]CEO, Applied Science Associates, and Temple University,
Philadelphia, Pennsylvania, USA.
[10]The Boeing Company, Huntington Beach, California.
[11]Alfvèn Laboratory, Royal Institute of Technology, Stockholm, S-100 44, Sweden.
[12]Alpha Foundation, Institute of Physics, 11 Rutafa Street, Building H, H-1165, Hungary.
[13]Smoluchowski Institute of Physics, Jagiellonian University,
ul Reymonta 4, Krakow, Poland.
[14]Indian Statistical Institute, Calcutta, India.
[15]Labo de Gravitation et Cosmologie Relativistes, Université Pierre et Marie Curie,
Tour 22-12, 4 ème étage, BP 142, 4 Place Jussieu, 75252 Paris Cedex 05, France.

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1 INTRODUCTION

In this brief comment it is shown straightforwardly that the B Cyclic Theorem[1]-[10] is Lorentz invariant by considering Lorentz boosts of the definition:

$$\mathbf{B}^{(3)} := -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{1}$$

where g is a constant and where $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ is a plane wave vector potential.

2 SIMPLE DEMONSTRATION OF INVARIANCE

If we accept definition (1), which is the fundamental definition of $\mathbf{B}^{(3)}$ in O(3) electrodynamics[1], a sub theory of the Sachs Einstein theory of electrodynamics[2] then its Lorentz invariance is proven straightforwardly as follows by considering Lorentz boosts in the X, Y and Z directions of the $\mathbf{B}^{(3)}$ field:

$$B_Z^{(3)\prime} = B_Z^{(3)} \tag{2}$$

$$B_Z^{(3)\prime} = \gamma B_Z^{(2)} + \gamma\beta E_X^{(3)} = \gamma B_Z^{(3)} \tag{3}$$

$$B_Z^{(3)\prime} = \gamma B_Z^{(2)} - \gamma\beta E_Y^{(3)} = \gamma B_Z^{(3)} \tag{4}$$

In Jackson's notation a Z boost of $\mathbf{A}^{(1)}$ for example leaves it unchanged:

$$\mathbf{A}^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\gamma\beta \\ 0 & 0 & -i\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} \mathbf{A}_X^{(1)} \\ \mathbf{A}_X^{(2)} \\ 0 \\ 0 \end{bmatrix} = \mathbf{A}^{(1)} \tag{5}$$

and since $\mathbf{A}^{(2)}$ is the complex conjugate of $\mathbf{A}^{(1)}$, a Z boost in free space results in:

$$(\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)})' \tag{6}$$

and leaves $\mathbf{B}^{(3)}$ invariant. The effect of a Y boost on $\mathbf{A}^{(1)}$ is as follows:

$$\mathbf{A}^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma & 0 & i\gamma\beta \\ 0 & 0 & 1 & 0 \\ 0 & -i\gamma\beta & 0 & \gamma \end{bmatrix} \begin{bmatrix} \mathbf{A}_X^{(1)} \\ \mathbf{A}_Y^{(1)} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_X^{(1)} \\ \gamma\mathbf{A}_Y^{(1)} \\ 0 \\ -i\gamma\beta\mathbf{A}_Y^{(1)} \end{bmatrix} \tag{7}$$

and using:

$$B_Z^{(3)} = -ig\epsilon_{(1)(2)(3)} A_X^{(1)} A_Y^{(2)} \tag{8}$$

it is found that:

$$\gamma \mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{9}$$

and the definition of $\mathbf{B}^{(3)}$ is again invariant. Using $\mathbf{B}^{(0)} = \kappa \mathbf{A}^{(0)}$[1] converts eqn.(1) into the B Cyclic Theorem and both are self consistently invariant under X, Y, and Z Lorentz boosts. Therefore $\mathbf{B}^{(3)}$ is a fundamental magnetic field.

3 DISCUSSION

The above is a very simple demonstration of the fact that the $\mathbf{B}^{(3)}$ field of O(3) electrodynamics is invariant under a Lorentz transformation. The demonstration is almost trivial. Dvoeglazov[11] has recently amplified this demonstration by using bivectors. The demonstration by Dvoeglazov may or may not be correct mathematically but there are several statements in his paper with which we agree or disagree as follows.

1. The $\mathbf{B}^{(3)}$ field is already an observable of interferometry and physical optics, and of the inverse Faraday effect[1],[5]-[10]. This corrects Dvoeglazov's[11] statement that $\mathbf{B}^{(3)}$ has a solid theoretical foundation but has not been observed.

2. The $\mathbf{B}^{(3)}$ field is the archetypical field of a higher symmetry form of electrodynamics, O(3) electrodynamics[1], which is a Yang Mills theory and automatically Lorentz covariant. O(3) electrodynamics is homomorphic with Barrett's SU(2) electrodynamics[3], obtained from topology and Clifford algebra. It follows that O(3) electrodynamics is also obtainable from Clifford algebra as well as from other independent approaches such as the theories by Sachs[2] and by Lehnert[4].

3. In O(3) electrodynamics the phase factor is a Wu Yang phase factor, and not the ordinary exponential phase factors used by Dvoeglazov[11]. The use of a Wu Yang phase factor in O(3) electrodynamics successfully describes severl anomalies inaccessible to U(1) or Maxwell Heaviside electrodynamics[1].

4. It is agreed with Dvoeglazov[11] that the early criticisms by Comay[12] and Hunter[13] are incorrect because these criticisms are based on the U(1) level, on which $\mathbf{B}^{(3)}$ does not exist. This is succinctly pointed out for example by Lehnert[4].

5. Dvoeglazov[11] sets up a straw man to knock over in his paper by considering arbitrarily a bi-vector made up of an electric and magnetic field and treating its Lorentz transformation. His conclusion is that relativity theory admits the B Cyclic Theorem, as is shown in Section 2, and we agree with this. We also agree that $\mathbf{B}^{(3)}$ can be a component of a four-vector[10]. However Dvoeglazov's claim[11] that the B cyclic equation is a relation between components of a magnetic field is erroneous. The fields $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ in general include all magnetic

field components. If plane waves, for example, are chosen for $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$, there are two scalar components, and $\mathbf{B}^{(3)}$ has one component in the Z axis, if this is chosen to be the axis of propagation.

6. Dvoeglazov[11] claims that the B Cyclic Theorem uses phase factors incompatible with his particular model, made up of bi-vectors. However the B Cyclic Theorem was not initially constructed from bi-vectors, so Dvoeglazov's criticism is misplaced and obscure. As shown in Section 2, bi-vectors are not necessary to prove the Lorentz invariance of the definition of $\mathbf{B}^{(3)}$ in O(3) electrodynamics.

Nevertheless Dvoeglazov's paper has elements of interest and may or may not be correct mathematically. However, it has nothing to do with Lorentz invariance of eqn.(1), which as we have shown, is proven trivially. M. W. Evans recalls some of the correspondence with Dvoeglazov to which the latter refers[11], and recalls that Dvoeglazov immediately accepted the B Cyclic Theorem enthusiastically, and developed its theory in ref.[10] without criticism. Therefore this latest paper by Dvoeglazov is cryptic and over-complicated.

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