ON THE NATURE OF THE $B^{(3)}$ FIELD

ABSTRACT

It is shown that the Maxwell-Heaviside theory is incomplete and limited, for example, it is unable to describe the Sagnac effect, either with platform at rest or in motion. An electrodynamical theory based on a physical internal O(3) gauge space is shown to provide the correct explanation in classical electrodynamics for the Sagnac effect, and for interferometry in general, for example Sagnac and Michelson interferometry. The U(1) sector of unified field theory must therefore be replaced by an SU(2) sector broken to O(3). This has numerous consequences for unified field theory.

INTRODUCTION

It is well known that the unmodified Maxwell-Heaviside theory of electrodynamics fails to describe the Sagnac effect, either with platform at rest or in motion {1-5}. It is less well known that it also fails to describe the Michelson effect, which is interference at a beam splitter caused by reflection from a mirror {6}. The reasons are simple, the d'Alembert equation in vacuo is invariant both under motion reversal symmetry (T) and parity inversion symmetry (P) on the classical level. In this paper, it is demonstrated that the recently inferred $\boldsymbol{B}^{(3)}$ field is the root cause of the phenomenon of interferometry, and that electrodynamics is an O(3) symmetry gauge field theory able to describe interferometry and other effects more self consistently than the received view, which is that classical electrodynamics is described by the Maxwell-Heaviside equations, which are invariant under a U(1) gauge transformation, and are Lorentz invariant, gauge invariant, and metric invariant in vacuo.

In Section 2, a prolegomenon is provided dealing with some recent incorrect assertions about the $\boldsymbol{B}^{(3)}$ field by Hunter {7} and by Comay {8}. In Section 3, it is shown that the Maxwell-Heaviside equations fail to describe the Sagnac and Michelson effects; and in Section 4, it is shown that these are due to the topological phase generated by an area integral over the $\boldsymbol{B}^{(3)}$ field, which thus provides the first correct understanding of interferometry. This is a major milestone in optics and spectroscopy.

MISCONCEPTIONS REGARDING THE $B^{(3)}$ FIELD.

The papers by both Hunter {7} and Comay {8} are entirely erroneous, indeed meaningless, because they apply a U(1) gauge field theory to the $\boldsymbol{B}^{(3)}$ field {9-21}, which has been developed by several authors independently. The $\boldsymbol{B}^{(3)}$ field is defined within an O(3) gauge field theory applied to classical electrodynamics with a physical internal space in the complex basis {9-21} ((1), (2), (3)), based on circular polarization. The paper by Comay {8} has been adequately answered already by Evans and Jeffers {22}, so we deal in this section with the paper by Hunter {7}. This paper is sequentially and entirely erroneous because it is written within the context of the Maxwell-Heaviside theory, invariant under a U(1) gauge transformation. The claims made there about $\boldsymbol{B}^{(3)}$ are therefore wholly incorrect, because the latter is defined in a theory of electrodynamics {9-21} which is covariant under an O(3) gauge transformation. The basic structure of this theory defines $\boldsymbol{B}^{(3)}$ as the component of a field tensor:

$$\boldsymbol{B}^{(3)*} \equiv -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \qquad (1)$$

where g is the coupling factor between source and field, and where $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is the conjugate product of physical and classical vector potentials. The product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is covariant under an O(3) gauge transformation, but is not invariant under a U(1) gauge transformation {9-21}. In a U(1) gauge theory applied to electrodynamics, the product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is indeed identically zero by definition, although this product is a physical observable {9-21}. In terms of magnetic flux density, eqn. (1) can be rewritten as:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*} \tag{2}$$

where $\boldsymbol{B}^{(3)}$ is defined as $B^{(0)}\boldsymbol{k}$, where $\boldsymbol{k}$ is the unit vector in the propagation direction. Eqn. (2) is written in the circular basis defined by the unit vectors {9-21}:

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) = \boldsymbol{e}^{(2)*}; \qquad \boldsymbol{e}^{(3)} = \boldsymbol{k} \tag{3}$$

and so eqn. (3) can be rewritten as:

$$B^{(0)2}\boldsymbol{k} = B^{(0)2}\boldsymbol{k} \tag{4}$$

Hunter {7} fails entirely to understand that eqn. (2) is one of O(3) gauge theory. It is clear from eqn. (4) that the magnitude of $\boldsymbol{B}^{(3)}$ is $B^{(0)}$ and that $\boldsymbol{e}^{(3)} = \boldsymbol{k}$. Hunter attempts to make an obscure and incorrect distinction between the object "$B^{(3)}\boldsymbol{e}^{(3)}$" and the object "$B_Z\boldsymbol{k}$". Eqn. (4) shows that:

$$B^{(0)} = B^{(3)} = B_Z \tag{5}$$

so the distinction that Hunter apparently attempts to make between $B^{(0)}$ and B_Z is spurious and erroneous. All his subsequent remarks based on this error are sequentially erroneous and to these authors, very obscure. In his section 3.4, there appears an irrelevant description of a model by Hunter and Wadlinger {23}, which is a model built up within the U(1) theory of electro-magnetism. The $\boldsymbol{B}^{(3)}$ field is defined, on the other hand, only within an O(3) gauge theory of electromagnetism, which we will refer to for brevity as "O(3) electrodynamics". In section 3.4.1, Hunter shows that the $\boldsymbol{B}^{(3)}$ field is non-zero in the Hunter Wadlinger model, but asserts that B_Z is zero within the same model. The calculations used by Hunter in this section were originally provided to him by M.W. Evans {23a}, but the source is not cited. Since B_Z is the magnitude of $\boldsymbol{B}^{(3)}$, the whole of section 3.4.1. of Hunter is self contradictory, spurious and obscure. Effectively, he calculates $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, finds it to be non-zero, and then states that it is zero. This is of course the flaw in U(1) theory that leads to O(3) electrodynamics, in which $\boldsymbol{B}^{(3)}$ is self-consistently defined.

His section 4 is a standard treatment of the third and other Stokes parameters as normally viewed within U(1) electrodynamics. In U(1), the third Stokes parameter {9-21} is simultaneously zero and non-zero, because in the U(1) gauge field structure, the third Stokes parameter is zero, but is used regularly as defined by Stokes phenomenologically to describe circular polarization. This is a fundamental self-inconsistency of classical electrodynamics viewed as a U(1) gauge theory. The state of confusion in this paper is such that the $\boldsymbol{B}^{(3)}$ is simultaneously asserted to be zero and also (in its conclusion) to be the third Stokes parameter, which is non-zero.

In section 4.1, there appears to be an attempt to redefine $\boldsymbol{B}^{(3)}$ in terms of what Hunter describes as "the entire electromagnetic field", again within the incorrect U(1) context. Effectively, this section sets out to define $\boldsymbol{B}^{(3)}$ in terms of a quantity in his eqn. (23) that already contains $\boldsymbol{B}^{(3)}$ and concludes that such a procedure will define $\boldsymbol{B}^{(3)}$ in terms of a quantity that already includes $\boldsymbol{B}^{(3)}$. This is impeccable logic, but physically meaningless. It is also not explained why this definition can exist if $\boldsymbol{B}^{(3)}$ is zero as asserted, but also simultaneously non-zero because it is the third Stokes parameter. So the confusion is compounded endlessly by the initial erroneous assumption that $\boldsymbol{B}^{(3)}$ is a component of U(1) electrodynamics. In section 4.3, it is asserted that $\boldsymbol{B}^{(3)}$ is a vector function of $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ but is zero, despite the fact that in the conclusion it is non-zero, being a rediscovery, according to Hunter, of the third Stokes parameter. In section 4.4., the U(1) Poynting Theorem is erroneously applied to $\boldsymbol{B}^{(3)}$. The correct way of defining the stress energy momentum tensor in O(3) is given elsewhere {24}. In section 4.5, the confusion becomes epidemic in proportion, it is stated by Hunter that Barron {25} defines $\boldsymbol{B}^{(3)}$ in terms of the third Stokes parameter, but that Lakhtakia {26} defines it as the inverse. The truth is that Barron asserted that $\boldsymbol{B}^{(3)}$ is zero {25} and was adequately answered by Evans {27}, and that Lakhtakia described $\boldsymbol{B}^{(3)}$ as "ghastly" {26}, again adequately answered by Evans {28}. It is not clear whether a ghastly $\boldsymbol{B}^{(3)}$ is physical or not and in consequence, we do not know what Lakhtakia means in terms of natural philosophy. In a book edited by Lakhtakia {9}, Barrett reaches the conclusion that $\boldsymbol{B}^{(3)}$ is non-zero, within an SU(2) gauge electrodynamics isomorphic with O(3). Therefore Hunter's scholarship is non-existent. None of the replies by Evans are quoted, so they are cited here for convenience.

In section (5), Hunter cites the work by Warren {29}, Rikken {30} and Raja et alia {31}, but omits the replies to this work which are given as follows for convenience {32-35}. The confusion is now pandemic because $\boldsymbol{B}^{(3)}$ is zero, non-zero, and zero again. The use of the U(1) Stokes Theorem by Comay {8}to prove that, indeed, $\boldsymbol{B}^{(3)}$ does not exist in U(1) is impeccable logic, but the fact of the matter is that $\boldsymbol{B}^{(3)}$ exists in O(3), not in U(1). It is already perfectly well known that it does not exist in U(1). The reply to Comay by Evans and Jeffers {22} is cited, perhaps by oversight, but no comment is offered by Hunter on this perfectly adequate reply except to state that "..the scenario is somewhat different...".

In its conclusion, the rest of this meaningless paper is undermined neatly by the assertion that $\boldsymbol{B}^{(3)}$ is a rediscovery of Stokes three. Now since Stokes three is non-zero, how can $\boldsymbol{B}^{(3)}$ be zero if it is a rediscovery of the same thing? We leave this as an exercise for the student.

THE FAILURE OF MAXWELL-HEAVISIDE THEORY IN INTERFEROMETRY.

It is shown in this section that the Sagnac and Michelson effects {1-6, 10} cannot be explained by the Maxwell-Heaviside theory, so a U(1) symmetry gauge theory cannot be the correct one for the electromagnetic sector of unified field theory, a major failure of the standard model. In the Sagnac effect, discovered ninety years ago, a beam of light is split by means of a combined beam splitter/interferometer and sent in opposite directions around the circumference of a disc using mirrors or optical fibers. An interference pattern is observed on the interferometer with platform at rest. This pattern is shifted to one side or another according to whether the platform is rotated anticlockwise (A) or clockwise (C). The effect is frame independent, and the same to an observer on or off the disc {1}.

This effect is a counter-example to Maxwell-Heaviside theory because the vacuum d'Alembert wave equation is invariant under motion reversal symmetry (T):

$$T(\Box A^{\mu} = 0) = (\Box A^{\mu} = 0) \tag{6}$$

Motion reversal symmetry generates the A loop from the C loop, and so the phase difference in Maxwell-Heaviside theory is zero, contrary to observation. Equivalently, the Maxwell-Heaviside equations in vacuo are invariant under T and so no phase shift should be observed with platform at rest. These equations also fail to describe the phase shift with platform in motion because they are metric invariant, and invariant under a U(1) gauge transformation, and so are invariant under frame rotation. No additional phase shift is predicted either for the A or C loops, contrary to observation. U(1) gauge transformation simply adds a random quantity to the electromagnetic phase of the Maxwell-Heaviside equations in vacuo, making the phase unphysical, or random. Finally, the four potential in this theory is asserted to be unphysical on the classical level. Several experimental counter-examples to this assertion have been given elsewhere {9-21} by several authors.

A review of the attempts to modify the Maxwell-Heaviside theory to accommodate the Sagnac effect has been given by Barrett {3}, who refers to an early experiment by Pegram {36} which shows that the true equations of electrodynamics in the vacuum cannot be invariant under a Lorentz transformation. The well known attempt by Post {3} to modify the Maxwell-Heaviside theory in vacuo is criticized by Barrett {10}, who finds a self-inconsistency in Post's argument. Post's solution was to modify the vacuum constitutive relations within the context of a U(1) theory, but as pointed out by Barrett {10}, it is not possible to incorporate the experimental result by Pegram {36} into the analysis without modifying the Maxwell-Heaviside equations themselves. This will be carried out in Section 4 and a simple method given to account for the Sagnac effect with a precision of one part in 10^{20}. The Sagnac effect is due to the $\boldsymbol{B}^{(3)}$ field, as is all interferometry, a major advance in understanding in optics, based on a higher symmetry gauge field theory applied to electrodynamics {9-21}. Both the Sagnac and Pegram effects have been ignored by the protagonists of unmodified Maxwell-Heaviside theory for nearly a century. The failure of these equations to describe interferometry is a major failure of the standard model, which is therefore in need of development.

The vacuum d'Alembert equation is also invariant under parity inversion, as are the Maxwell-Heaviside equations:

$$P(\square A^{\mu} = 0) = (\square A^{\mu} = 0) \tag{7}$$

In the Michelson interferometer, a beam of light is sent from a beam splitter to a mirror and back to the beam splitter. The reversal of the beam's direction is generated by parity inversion, so in the Maxwell-Heaviside theory, the electromagnetic phase is unchanged by the round trip of the light beam from beam splitter to mirror and back. The other beam generated by the beam splitter similarly travels to a mirror at right angles to the first mirror, and similarly arrives back at the beam splitter with phase unchanged. The two phases are unchanged irrespective of the distance between the beam splitter and mirror, so in Maxwell-Heaviside theory, there is no interference pattern, or interferogram. This is obviously contrary to observation, the Michelson interferometer being the basis for all Fourier transform infrared spectrometers {6}.

This is a second major counter-example to unmodified Maxwell-Heaviside theory and to the standard model, which incorporates a U(1) electromagnetic sector symmetry. We arrive at the major conclusion that a gauge invariant theory of electrodynamics cannot describe interferometry. In the next section, a gauge covariant theory {9-21}, based on a physical internal gauge symmetry, is used to explain the Sagnac and Michelson effects straightforwardly in terms of the $\boldsymbol{B}^{(3)}$ field of electromagnetic radiation. The $\boldsymbol{B}^{(3)}$ field is responsible for optical interferometry and is obviously a physical observable.

EXPLANATION OF THE SAGNAC AND MICHELSON EFFECTS WITH O(3) ELECTRODYNAMICS.

To explain the phase difference observed with platform at rest in the Sagnac effect {1}, we start from eqn. (1) and use the identity:

$$\pi R^2 = \int 2\pi R dR = \iint dAr \tag{8}$$

where R is the radius of a circular Sagnac platform, e.g. an optical fiber. Multiply both sides of this identity by:

$$B^{(0)} = \kappa A^{(0)} \tag{9}$$

to obtain

$$2\pi\kappa A^{(0)} \int \boldsymbol{R} \cdot d\boldsymbol{R} = \iint \boldsymbol{B}^{(3)} \cdot d\boldsymbol{Ar} \tag{10}$$

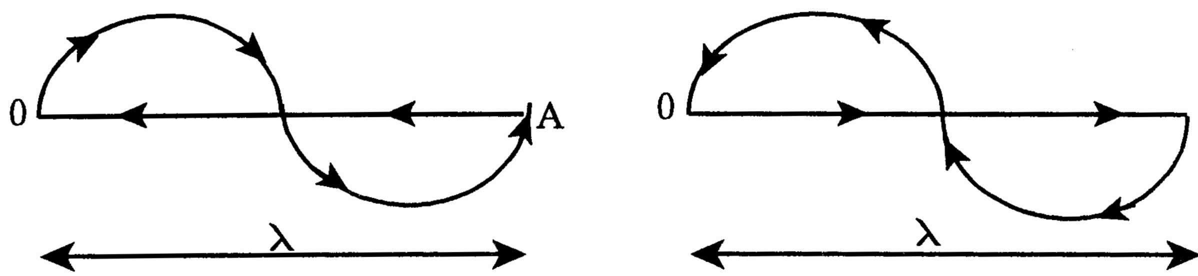
and multiply both sides again by the factor g to give the phase {9, 10, 20}:

$$\phi = 2\pi\, g\kappa A^{(0)} \int \boldsymbol{R} \cdot d\boldsymbol{R} = g \iint \boldsymbol{B}^{(3)} \cdot d\boldsymbol{Ar} \tag{11}$$

Since $g = \kappa / A^{(0)}$, it is easily checked that eqn. (11) is the identity:

$$\phi = \kappa^2 \int 2\pi\, \boldsymbol{R} \cdot d\boldsymbol{R} = \kappa^2 \iint \boldsymbol{k} \cdot d\boldsymbol{Ar}. \tag{12}$$

Consider electromagnetic radiation propagating around the Sagnac ring. It is seen that the ordinary integral on the left hand side of eqn. (11) can be replaced by a line integral. For one wavelength, the line integral is sketched as follows:



The contributions to the line integral off the axis OA or AO is zero. For a transverse plane wave for example, we obtain:

$$\oint \text{offaxis} = -\int_0^{2\pi} \sin\phi\, d\phi\, \boldsymbol{i} + \int_0^{2\pi} \cos\phi\, d\phi\, \boldsymbol{j} = \boldsymbol{0}. \tag{13}$$

Eqn. (12) therefore becomes:

$$\phi = \kappa^2 \oint 2\pi\, \boldsymbol{R} \cdot d\boldsymbol{R} = \kappa^2 \iint \boldsymbol{k} \cdot d\boldsymbol{Ar} \tag{14}$$

If we consider a segment of the Sagnac ring consisting of one wavelength of the electromagnetic radiation propagating through the ring, we have:

$$R = x\lambda = \frac{2\pi x}{\kappa}. \tag{15}$$

The final formula for the phase of the Sagnac interferometer is therefore:

$$\phi = 4\pi^2 \oint \kappa \cdot d\boldsymbol{R} = g \iint \boldsymbol{B}^{(3)} \cdot d\boldsymbol{Ar} = \kappa^2 Ar \tag{16}$$

with platform at rest. The phase ϕ defined in eqn. (16) has the following properties under T and P:

$$T(\phi) = -\phi; \qquad P(\phi) = -\phi \tag{17}$$

because of the property of a line integral, which changes sign for traversal in opposite directions. The phase difference seen with platform at rest between A and C loops is 2ϕ, and the interferogram is:

$$\phi = \cos(2(\phi \pm \pi)). \tag{18}$$

Eqn. (16) equates the dynamical (left hand side) and topological (right hand side) phases by identity.

Now rotate the platform about an axis Z perpendicular to its plane. In O(3) electrodynamics, a change in phase means a change in the orientation in the internal gauge space, which in this case is the physical three dimensional space. A rotation of the platform should therefore result in a change in phase {9-21}, as observed {1}. The rotation is represented by the rotation of the n-dimensional vector field ψ {10} of general gauge field theory:

$$\psi' = e^{iJ_z\alpha}\psi \tag{19}$$

where J_Z is the Z axis rotation generator, and α an angle in the plane of the Sagnac platform. The rules of gauge transformation in O(3) electrodynamics show {20}that as a result of this rotation:

$$\begin{aligned} \kappa &\rightarrow \kappa \pm \Omega / c \\ \omega &\rightarrow \omega \pm \Omega \end{aligned} \tag{20}$$

where $\Omega = \partial\alpha/\partial t$ is the angular frequency of the rotating platform. This causes a change {20} in the phase (A - C):

$$\Delta\phi = \frac{4\omega\Omega Ar}{c^2} \tag{21}$$

Defining $\Delta\phi = \omega\Delta t$, we obtain the Sagnac formula {1,2}:

$$\Delta t = \frac{4\Omega Ar}{c^2}. \tag{22}$$

Therefore the standard concepts of O(3) gauge theory applied to classical electrodynamics lead directly to the Sagnac formula.

In Michelson interferometry, the properties of the phase, eqn. (17), again apply, and a phase difference occurs at the beam splitter, resulting in the familiar interferogram. The dynamical and topological phases are identified in the same manner as in Sagnac interferometry, and indeed, the topological phase due to $B^{(3)}$ has been observed in all types of interferometry {10}. It is concluded that the $B^{(3)}$ field is responsible for interferometry.

CONCLUSION

A correct explanation of all interferometric phenomena in optics requires a gauge covariant theory such as O(3) electrodynamics. A gauge invariant theory such as Maxwell-Heaviside theory fails to explain Sagnac and Michelson interferometry. This strongly suggests that electrodynamics is a gauge theory with internal symmetry O(3); or more generally, a gauge theory with internal symmetry SU(2) broken to O(3). This is the first correct explanation in classical electrodynamics of the phenomenon of interferometry. The empirical evidence for this conclusion is overwhelming.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge many internet discussions with colleagues internationally, and to acknowledge funding to institutional members of AIAS (Alpha Foundation's Institute of Advanced Study).

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