[27] S. W. Homans, *A Dictionary of Concepts in NMR* (Clarendon, Oxford, 1989).
[28] R. R. Ernst, G. Bodenhausen and A. Wokaun, *Principles of Nuclear Magnetic Resonance in One and Two Dimensions* (Oxford University Press, Oxford, 1987).
[29] M. W. Evans, G. J. Evans, W. T. Coffey, and P. Grigolini, *Molecular Dynamics* (Wiley-Interscience, New York, 1982).
[30] described in W. Heitler, *The Quantum theory of Radiation*, 3rd. edn. (Oxford University Press, Oxford, 1954).
[31] P. A. M. Dirac, *Lectures on Quantum Field Theory* (Belfer Graduate School, Yeshiva University, New York, 1966).

Paper 15

The Microwave Optical Zeeman Effect Due to $\boldsymbol{B}^{(3)}$

The optical conjugate product of a circularly polarized laser is used in the Dirac equation to show the presence of a microwave frequency optical Zeeman effect which is proportional at a given angular frequency to the Evans-Vigier field $\boldsymbol{B}^{(3)}$ of the microwave radiation. An experimental arrangement to detect this effect is proposed, using ESR technique.

Key words: Microwave optical Zeeman effect, $\boldsymbol{B}^{(3)}$ field.

15.1 Introduction

Recently, it has been demonstrated that the Dirac equation of one electron in a circularly polarized electromagnetic field can be solved to show the existence of the $\boldsymbol{B}^{(3)}$ (Evans-Vigier) field, a magnetic flux density whose classical source is the conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ of plane wave solutions of Maxwell's equations in the vacuum. In the appropriate circular basis [1—5] there exist the cyclically symmetric relations between fields,

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \text{ et cyclicum}, \tag{2.15.1}$$

so that $\boldsymbol{B}^{(3)}$ is phase free. Here $B^{(0)}$ is a scalar amplitude (tesla). Whenever radiation magnetizes matter, the effect depends on $\boldsymbol{B}^{(3)}$ at first and second order [6]. At visible frequencies, there is an inverse Faraday effect and optical Zeeman effect which are linear [7] in the beam power density, I, (in watts per square meter). These effects originate in $iB^{(0)}\boldsymbol{B}^{(3)*}$. At microwave frequencies and sufficient power densities, however, the inverse Faraday effect becomes proportional directly [8—10] to the beam's $\boldsymbol{B}^{(3)}$ field, and thus to $I^{1/2}$. Since $\boldsymbol{B}^{(3)}$ travels at the speed of light in vacuo, and cannot exist in isolation of its source, the conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, there is no free Faraday induction, i.e., a modulated laser sent through an induction coil without a sample will produce no signal in the coil. The reason for this is that $\boldsymbol{B}^{(3)}$ travels at c in the vacuum and under these conditions the only electric fields allowed by symmetry and relativity are the ordinary, transverse, plane waves $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$, these being complex conjugate pairs in the basis (1), (2), (3).

In this Letter, the Dirac equation is used to show that there exists a microwave frequency optical Zeeman effect which for a given microwave pump frequency ω is directly proportional to the $\boldsymbol{B}^{(3)}$ field of the radiation. In Sec. 15.2, the Dirac equation is solved for the interaction of the beam conjugate product with one electron. At visible frequencies this produces the optical Zeeman effect, which is proportional to the beam power density I. At microwave frequencies the Dirac equation produces an optically induced Zeeman effect proportional to the square root of I. At these frequencies the beam property producing the effect is $\boldsymbol{B}^{(3)}$. Section 15.3 is a discussion of this result in terms of transfer of photon angular energy to the electron, and it is shown that the result of the calculation in Sec. 15.2 is consistent with conservation of angular energy in a photon-electron collision.

15.2 The Optical Zeeman Effect from the Dirac Equation

Since $\boldsymbol{B}^{(3)}$ cannot exist in isolation of its source, the electromagnetic conjugate product (Eq.(2.15.1)), we calculate the optical Zeeman effect from the pure electromagnetic term in the Dirac equation [6] of an electron in a circularly polarized electromagnetic beam,

$$\hat{W}u = \left(\frac{ie^2}{2m_0 + eA^{(0)}/c} \sigma \cdot \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \right) u. \tag{2.15.2}$$

Here u is a Dirac four-spinor in the standard representation [6], $\hat{W}$ is an energy eigenoperator whose eigenvalue is given within brackets on the right hand side. In Eq. (2.15.2) e is the charge on the electron, σ a Pauli matrix, m_0 the electron mass, and c the speed of light in vacuo. The conjugate product is expressed as $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, where $\boldsymbol{A}^{(1)}$ is a vector potential plane wave [6] and $\boldsymbol{A}^{(2)}$ its complex conjugate. The scalar amplitude of $\boldsymbol{A}^{(1)}$ is $A^{(0)}$, and the minimal prescription [6] has been used to describe the momentum and energy imparted relativistically to the electron by the field.

Equation(2.15.2) contains no reference to any field free electron momentum, and uses the rest frame approximation [6] $En \sim m_0c^2$ for the electron energy. Non-relativistically, therefore, there would be no electron energy in the absence of the beam. The equation also assumes that the scalar potential is $A^{(0)}/c$, and not zero as in the Coulomb gauge. This means physically that the beam imparts energy to the electron as well as momentum. Such a picture is compatible with the Lorentz gauge [6] and a manifestly covariant A_μ four-vector.

From Eq.(2.15.2), the Hamiltonian expectation value (i.e., the energy eigenvalue) is,

$$\langle H \rangle = \frac{ie^2c}{2m_0c + eA^{(0)}} \sigma \cdot \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{2.15.3}$$

which is the relativistic expression of the optical Zeeman effect. Here σ is a Pauli matrix, so we are dealing with the relativistic half-integral spin of the electron in a classically expressed field. The optical Zeeman effect at visible and microwave frequencies emerges by a simple consideration of limits as follows.

(1) When the electron rest momentum is much greater than that imparted by the beam to the electron,

$$2m_0 c \gg eA^{(0)}, \tag{2.15.4}$$

the energy eigenvalue becomes,

$$<H> \rightarrow i\frac{e^2}{2m_0}\sigma \cdot \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{2.15.5}$$

which is to order I, i.e., proportional to the beam power density. Using,

$$A^{(0)} = \frac{B^{(0)}}{\kappa} = \frac{c}{\omega}B^{(0)}, \tag{2.15.6}$$

the condition (4) becomes,

$$\omega \gg \frac{e}{2m_0}B^{(0)}, \tag{2.15.7}$$

which is satisfied at *visible* frequencies for all but enormous, unattainable I. At visible frequencies the Zeeman shift is therefore twice the energy in Eq.(2.15.5), which can be expressed as,

$$<H> \rightarrow -\frac{e^2 c^2}{2m_0\omega^2}\sigma \cdot B^{(0)}\boldsymbol{B}^{(3)}. \tag{2.15.8}$$

The term,

$$\chi' := -\frac{e^2 c^2}{2m_0\omega^2}, \tag{2.15.8a}$$

is the one electron susceptibility [6].

(2) In the opposite limit, when the momentum imparted by the beam to the electron is much greater than the electron rest momentum,

$$eA^{(0)} \gg 2m_0 c, \tag{2.15.9}$$

Eq.(2.15.3) reduces to,

$$<H> \rightarrow i\frac{ec}{A^{(0)}}\sigma \cdot \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{2.15.10}$$

The limit(2.15.9) can be rewritten as

$$\omega \ll \frac{e}{2m_0}B^{(0)}, \tag{2.15.11}$$

which is attainable with *microwave* pulses of high power density [11]. Using in Eq.(2.15.10) the relation [6],

$$\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} = iA^{(0)2}\boldsymbol{e}^{(3)}, \tag{2.15.12}$$

the energy becomes,

$$<H> \underset{m_0 \rightarrow 0}{\rightarrow} -ec\sigma \cdot A^{(0)}\boldsymbol{e}^{(3)} = -\frac{ec^2}{\omega}\sigma \cdot \boldsymbol{B}^{(3)}, \tag{2.15.13}$$

which for a given angular frequency, ω, is proportional to $\boldsymbol{B}^{(3)}$ of the beam and therefore [11] to the square root of its power density. Equation(2.15.13) shows that the optical Zeeman effect at microwave frequencies is determined entirely by two beam properties, ω and $\boldsymbol{B}^{(3)}$.

15.3 Discussion

Equation(2.15.13) can be rewritten as

$$\langle H \rangle \underset{m_0 \to 0}{\to} -ceA^{(0)}\sigma \cdot \boldsymbol{e}^{(3)}, \tag{2.15.14}$$

which is seen to have the correct units of energy (because $\sigma \cdot \boldsymbol{e}^{(3)}$ is unitless) and because $eA^{(0)}$ is electron momentum magnitude acquired from the beam.

Using the charge quantization condition [6,12], which is implied by the existence of $\boldsymbol{B}^{(3)}$ [12],

$$eA^{(0)} = \hbar\kappa, \tag{2.15.15}$$

Eq.(2.15.14) becomes,

$$\langle H \rangle \underset{m_0 \to 0}{\to} -\hbar\omega\sigma \cdot \boldsymbol{e}^{(3)}, \tag{2.15.16}$$

and this shows that the rotational energy, $\hbar\omega$, of the photon has been transferred completely to the electron. In this limit, the expected Zeeman splitting is therefore $2\hbar\omega$. This limit can never be attained in practice because the rest momentum of the electron is always non-zero in special relativity, but under condition(2.15.11), it can be approximated.

The equation(2.15.2), which starts from the conjugate product of the classical field, has therefore produced the result expected on the grounds of

conservation of rotational energy in a perfectly elastic collision between a photon and an electron. This result was attained through the condition(2.15.11), and is consistent with the fact that the photon has lost $\hbar\omega$ and the electron has gained $\hbar\omega$ in this limit. The electron has absorbed the photon and acquired the photon spin. Since the electron has spin states determined by the matrix σ, the spin up state acquires energy $\hbar\omega$, and the spin down state acquires $-\hbar\omega$ for the same sense of circular polarization in the beam (i.e., left or right).

It is to be noted that if $\boldsymbol{B}^{(3)} =? \ \boldsymbol{0}$ then this photon absorption process cannot occur at microwave or at visible frequencies. Thus $\boldsymbol{B}^{(3)}$ is a fundamental property of the beam which cannot exist, however, in isolation of its source, the beam conjugate product, which was the optical property used as the starting point of our calculation in Eq.(2.15.2).

It should be possible to see the Zeeman splitting $2\hbar\omega$ by tuning a microwave probe beam to about the frequency 2ω, using ESR technique [13,14]. One configuration which can be suggested is to send an electron or atomic beam in the Z axis, a microwave pump pulse at frequency ω in Y and a microwave probe pulse at about 2ω synchronized with the pulse. The probe should be tunable around 2ω because this is an ideal condition as just discussed.

Acknowledgments

This paper emerged from discussions at a seminar given at York University, Toronto, Canada with several colleagues of the Department of Physics there, and following an invitation by Prof. Stanley Jeffers. These discussions are much appreciated.

References

[1] M. W. Evans, *Physica B* **182**, 227, 237 (1992); **183**, 103 (1993); **190**, 310 (1993).

[2] M. W. Evans, *The Photon's Magnetic Field.* (World Scientific, Singapore, 1992).
[3] M. W. Evans, and S. Kielich, eds., *Modern Nonlinear Optics*, Vols. 85(1), 85(2), 85(3) of *Advances in Chemical Physics,* I. Prigogine and S. A. Rice, eds., (Wiley Interscience, New York, 1993).
[4] M. W. Evans in A. van der Merwe and A. Garuccio, eds., *Waves and Particles in Light and Matter.* (Plenum, New York, 1994), the de Broglie Centennial.
[5] A. A. Hasanein and M. W. Evans, *Quantum Chemistry.*, vol. 1, *The Photomagneton in Quantum Field Theory.* (World Scientific, Singapore, 1994).
[6] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Volume 1: The Field* $\boldsymbol{B}^{(3)}$, (Kluwer Academic Publishers, Dordrecht, 1994); ibid., *The Enigmatic Photon, Volume 2: Non-Abelian Electrodynamics*, (Kluwer Academic Publishers, Dordrecht, 1995); *The Enigmatic Photon, Volume 3, A Symposium in Honour of Jean-Pierre Vigier*, conference volume, August 28th to 30th, 1995, at York University, Toronto, in preparation.
[7] P. S. Pershan, *Phys. Rev.* **130**, 919 (1963); J. P. van der Ziel, P. S. Pershan, and L. D. Malmstrom, *Phys. Rev.* **143**, 574 (1966); J. Deschamps, M. Fitaire, and M. Lagoutte, *Phys. Rev. Lett.* **25**, 1330 (1970).
[8] M. W. Evans, *Found. Phys.* **24**, 892, 1519, 1671 (1994).
[9] M. W. Evans, *Found. Phys. Lett.* **7**, 67, 209 (1994).
[10] M. W. Evans, ibid., in press.
[11] S. Roy, *Phys. Lett. A* in press; S. Jeffers, S. Roy and M. W. Evans, *Phys. Lett. A* in prep.; O. Costa de Beauregard, *Found. Phys. Lett.*, in press.
[12] M. W. Evans, *Found. Phys. Lett.*, in press.
[13] P. W. Atkins, *Molecular Quantum Mechanics,* 2nd. edn. (Oxford, 1982).
[14] K. A. Earle, Cornell University, Department of Chemistry, communications.

Paper 16

$\boldsymbol{B}^{(3)}$ Echoes

It is shown that the $\boldsymbol{B}^{(3)}$ field of vacuum electromagnetism regenerates itself throughout spacetime from repeated gauge transforms. These $\boldsymbol{B}^{(3)}$ *echoes* are physical magnetic fields which can be detected experimentally in principle through optical analogues of the Aharonov-Bohm effect.

Key words. Optical Aharonov-Bohm effect, action at a distance, $\boldsymbol{B}^{(3)}$ field

16.1 Introduction

The existence of the $\boldsymbol{B}^{(3)}$ field is established [1—12] by that of magneto-optic effects typified by the well-verified [13—21] inverse Faraday effect. In this note it is argued that the field is echoed throughout space-time by repeated gauge transformations into the vacuum of $\boldsymbol{A}$, where,

$$\boldsymbol{B}^{(3)} := \nabla \times \boldsymbol{A} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{2.16.1}$$

defines the original $\boldsymbol{B}^{(3)}$ in vacuo in a local region of space-time. Here $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ is a plane wave potential, a solution of the d'Alembert wave