Paper 5

# Fundamental Definitions for the Vacuum $\boldsymbol{B}^{(3)}$ Field

The fundamental definitions of the vacuum $\boldsymbol{B}^{(3)}$ field are developed in terms of the universal constants and radiation properties. The vacuum $\boldsymbol{B}^{(3)}$ field is the expectation value of the photomagneton operator $\hat{\boldsymbol{B}}^{(3)}$, an irremovable and fundamental property of the vacuum electromagnetic field.

## 5.1 Introduction

In the received view of electromagnetism in vacuo [1—3], the fields are transverse to the direction of propagation, and the photon is massless. Recently, this view has been challenged at the fundamental level by the proposal of the $\boldsymbol{B}^{(3)}$ (longitudinal) component, generated by the conjugate product of the transverse fields, a component which is phase free [4—10]. The existence of $\boldsymbol{B}^{(3)}$ is shown by the class of inverse Faraday induction phenomena [11—16], typified by the inverse Faraday effect, magnetization by radiation. Further experimental support for its existence would become available from the $I^{1/2}$ induction profile expected from $\boldsymbol{B}^{(3)}$ [17] at radio frequencies. Here $I$ is the beam intensity in watts $m^{-2}$.

In this note, fundamental definitions of the vacuum $\boldsymbol{B}^{(3)}$ field are developed in terms of the universal constants and of fundamental radiation properties. The $\boldsymbol{B}^{(3)}$ field is defined as the irremovable and phase free expectation value of the photomagneton operator $\hat{\boldsymbol{B}}^{(3)}$ of one photon of energy $\hbar\omega$, where $\hbar$ is Dirac's constant and where $\omega$ is the angular frequency. This inference has recently been confirmed [18] by Muñera and Guzmán, who have shown the existence of a new class of longitudinal solutions in vacuo of the Maxwell equations. These authors isolated a component of their novel solutions which is phase independent and irremovable, thus confirming the earlier inference [4] that the photomagneton is a novel fundamental property of the photon and electromagnetic wave.

The monographs now available on $\boldsymbol{B}^{(3)}$ theory develop didactically the earliest theory [4—6], and clarify several aspects, linking up with work such as that of Hunter and Wadlinger [19] and Moles and Vigier [20]. It has been shown that the $\boldsymbol{B}^{(3)}$ field is defined in the vacuum by a component product of vector potentials $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$,

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{2.5.1}$$

where $e$ is the charge quantum [4—10]. The mode of interaction of $\boldsymbol{B}^{(3)}$ with a fermion is determined by this definition through the Dirac equation. It is significant that $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ emerges from the Dirac equation itself [7] and is no longer phenomenological, as in the earliest papers [4—6]. The Dirac-Pauli and Hamilton-Jacobi equations can therefore be used to show the expected $I^{1/2}$ dependence of inverse induction due to $\boldsymbol{B}^{(3)}$; providing a route to empirical detection of $\boldsymbol{B}^{(3)}$ at first order. In general, inverse Faraday induction belongs to the class of non-linear optical phenomena [11—16], and depends on a non-linear optical property [4—10],



$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \text{ et cyclicum,} \tag{2.5.2}$$

where $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ is the transverse magnetic component of, for example, a plane wave, in the circular basis ((1), (2), (3)). In Eq. (2.5.2), $B^{(0)}$ is the scalar magnitude of $\boldsymbol{B}^{(1)}$, and $\boldsymbol{B}^{(3)}$ is longitudinal and phase free. Experimental detection of $\boldsymbol{B}^{(3)}$ can therefore be achieved by showing the existence of the left hand side in Eq. (2.5.2). Effectively, this demonstration has been carried out, with the wisdom of hindsight, several times [11—16]. In the pre-1992 view, however, the existence of $\boldsymbol{B}^{(3)}$ through Eq. (2.5.2) was unknown, and the left hand side was constructed phenomenologically and known as the *conjugate product*.

Equation (2.5.1) has been developed recently within the framework of general relativity, using the inference [19] that the vacuum plane wave has a scalar curvature $R$, which in special relativity is not considered. (Curvatures and affine connections in Galilean space-time are zero by definition.) If the world-line of the charge quantum e is regarded as the fiducial geodesic in general relativity (a geodesic whose spatial trajectory is helical [19]) then Eq. (2.5.1) emerges from the Riemann tensor's antisymmetric contraction [7], giving a new equivalence principle for electromagnetism. If $\boldsymbol{B}^{(3)}$ is not considered, then a rigorously non-zero part of the Riemann curvature tensor disappears, the part that is quadratic in the affine connection. This inference opens new doors in field unification.

## 5.2 Fundamental Definitions in *S.I.* Units

In *S.I.* Units, the vacuum permeability is [21],

$$\mu_0 = 4\pi \times 10^{-7} Js^2C^{-2}m^{-1}, \tag{2.5.3}$$

and beam intensity, or power density, is measured as

$$I = \frac{c}{\mu_0}B^{(0)2} = cU_V, \tag{2.5.4}$$

where $B^{(0)} = |\boldsymbol{B}^{(3)}|$ and $U_V$ is radiation energy per unit volume ($Jm^{-3}$). The units of $B^{(0)}$ are tesla ($T$) = $Wb\,m^{-2} = JsC^{-1}m^{-2}$. For the conventionally massless photon, $c$ is the speed of light in $ms^{-1}$. Therefore the magnitude of the photomagneton is a magnetic flux (Wb) per unit area. From Eq. (2.5.4),

$$B^{(0)} = \left(\frac{\mu_0}{c}I\right)^{1/2} = (\mu_0 U_V)^{1/2}, \tag{2.5.5}$$

and is a conserved quantity in vacuo, being directly proportional to the *square root* of beam intensity. Under the right conditions [4—10], inverse induction due to $\boldsymbol{B}^{(3)}$ is also proportional to $I^{1/2}$, revealing the existence of $\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)}$.Here, $\boldsymbol{e}^{(3)}$ is a unit vector in the direction of propagation of the beam.

From Eq. (2.5.1),

$$B^{(0)} = \frac{e}{\hbar}A^{(0)2}, \tag{2.5.6}$$

where $A^{(0)} = |\boldsymbol{A}^{(1)}| = (\boldsymbol{A}^{(1)}\cdot\boldsymbol{A}^{(2)})^{1/2}$ in $JsC^{-1}m^{-1}$. Therefore,

$$A^{(0)2} = \left(\left(\frac{\mu_0}{c}\right)^{1/2}\frac{\hbar}{e}\right)I^{1/2}. \tag{2.5.7}$$

It is also known that $A^{(0)}$ and $B^{(0)}$ are related by the Maxwellian definition [21] of $\boldsymbol{A}$, i.e., $\boldsymbol{B} = \nabla\times\boldsymbol{A}$; and if $\boldsymbol{A}$ is taken to be a plane wave solution of the d'Alembert equation in vacuo, it follows [4—6] that



$$B^{(0)} = \kappa A^{(0)}, \tag{2.5.8}$$

where $\kappa$ is the wavevector. For the conventional massless photon; $\kappa = \omega/c$. From Eqs. (2.5.6) and (2.5.8) emerges *the minimal prescription for the free photon* [4—10].

$$eA^{(0)} = \hbar\kappa, \tag{2.5.9}$$

an equation which balances the classical momentum per photon $eA^{(0)}$, with its quantum equivalent $\hbar\kappa$. In terms of the photon energy (the quantum of energy),

$$\hbar\omega = ecA^{(0)}. \tag{2.5.10}$$

For the sake of argument, we have accepted the idea of a massless photon in deriving Eq. (2.5.10) from Eq. (2.5.9). In contravariant notation, Eqs. (2.5.9) and (2.5.10) imply that the momentum/energy of the free photon is,

$$\begin{aligned} p^\mu &:= (En/c, \boldsymbol{p}) = \hbar\kappa^\mu := (\hbar\kappa, \hbar\boldsymbol{\kappa}) \\ &= eA^\mu := e\left(A^{(0)}, \boldsymbol{A}\right). \end{aligned} \tag{2.5.11}$$

Note that the relativistically correct result in Eq. (2.5.11) is incompatible with the transverse gauge [22], in which it is *assumed* that vacuum solutions of the d'Alembert equation have no longitudinal or time-like components. As shown by Muñera and Guzmán [18], *this assumption is incorrect*, there exists a class of longitudinal solutions under well-defined conditions more general than that of the transverse gauge. Therefore, as shown experimentally in the Aharonov-Bohm effects [23], $A^\mu$ is a physical observable, not a mathematical convenience. Within a factor $e$, $A^\mu$ is simply the energy momentum $p^\mu$ of the free photon, a gauge *invariant* physical observable. This in turn suggests that there is the need for a wave equation in the vacuum which restricts gauge freedom. An example is the Proca equation [4], which uses a very small, but non-zero, photon mass on which currently available experimental data put an upper bound [24]. Since massless particles conventionally [25] have only transverse degrees of polarization, the Proca equation is also implied by and compatible with $\boldsymbol{B}^{(3)}$ [4—10].

From Eq. (2.5.4), the energy per unit volume for one photon (the quantum of electromagnetic energy, $\hbar\omega$) is,

$$\frac{\hbar\omega}{V} = \frac{1}{\mu_0} B^{(0)2}, \tag{2.5.12}$$

where $B^{(0)}$ is the magnitude of the photomagneton, the quantum of magnetic flux density, and $V$ is the average volume occupied by the photon. As shown by Hunter and Wadlinger [19], this is in general the volume of an ellipsoid, and in order to define this volume, the photon can be considered to be a wavicle, and not a particle. We therefore have three equations linking $A^{(0)}$ and $B^{(0)}$,

$$B^{(0)} = \frac{e}{\hbar} A^{(0)2}, \tag{2.5.13}$$

$$B^{(0)} = \frac{\omega}{c} A^{(0)}, \tag{2.5.14}$$

$$B^{(0)2} = \frac{\mu_0 e c}{V} A^{(0)}, \tag{2.5.15}$$

revealing the intricate inter-relations of basic vacuum electrodynamics.



A number of fundamental relations can now be derived from these three equations, in which $\hbar$, $\mu_0$, $c$ and $e$ are universal constants and in which $A^{(0)}$, $B^{(0)}$, $\omega$ and $V$ are electrodynamic quantities. From Eq. (2.5.13) in (2.5.15),

$$A^{(0)} = \frac{e\mu_0 c^3}{V\omega^2}, \tag{2.5.16a}$$

$$B^{(0)} = \frac{e\mu_0 c^2}{V\omega}. \tag{2.5.16b}$$

From Eq. (2.5.15) in (2.5.16b)

$$B^{(0)3} = \frac{e\mu_0^2 c^2 \hbar}{V^2}. \tag{2.5.17}$$

From Eq. (2.5.13) in (2.5.15),

$$A^{(0)} B^{(0)} = \frac{\mu_0 \hbar c}{V}. \tag{2.5.18}$$

From Eq. (2.5.14) in (2.5.18),

$$A^{(0)3} = \frac{\mu_0 c \hbar^2}{e} \frac{1}{V}. \tag{2.5.19}$$

From Eq. (2.5.16a) in (2.5.19) and (2.5.16b) in (2.5.17),

$$V = \frac{e^2 \mu_0 c^4}{\hbar} \frac{1}{\omega^3} . \tag{2.5.20}$$

Throughout these equations $A^{(0)}$ appears as a physical quantity, not a supplementary mathematical variable. For example, Eq. (2.5.19) shows that $A^{(0)3}$ is inversely proportional to the volume $V$ occupied by the photon of energy $\hbar\omega$, and $A^{(0)3}$ in consequence is as *physical* as $V$. An even more striking illustration of the physical nature of $A^{(0)}$ emerges from combining Eqs. (2.5.19) and (2.5.20) to give Eq. (2.5.10), which shows that $A^{(0)}$ is directly proportional to the observable $\omega$.

Equation (2..5.20) shows that for any finite frequency, $V$ is non-zero, meaning that the photon must always occupy a finite, frequency dependent, volume. It is a *point particle* only when $\omega$ is infinite, and at low enough frequencies, the volume $V$ becomes macroscopic (e.g. order of $km^3$). There are obvious difficulties in continuing to accept the picture of a photon as an elementary particle of nuclear dimensions, for example. These have been carefully discussed by Hunter and Wadlinger [19], who also report experimental data on the finite volume of the photon as wavicle rather than particle. It is well known that de Broglie and Einstein attacked these difficulties using the empty wave hypothesis [24] and by locating all of the mass of the photon near its core, the rest being wave-like in nature. The received view [22] prohibits photon mass, so that at low enough frequencies we are asked to accept the existence of the photon as an elementary particle with no mass, but with macroscopic dimensions. Experiments on the radius and volume of the photon [19] should surely be used to test this counter-intuitive view.

An insight to the physical meaning of the relations between $B^{(0)}$ and $A^{(0)}$, Eqs. (2.5.13) to (2.5.15), can be obtained from the fact that $ecA^{(0)}$ has the dimensions of $J$ (energy), and that $ec^2B^{(0)}$ has the dimensions of J $s^{-1}$ or W (power). The latter is dimensionally the product of energy $(ecA^{(0)})$ and frequency $(\omega)$. Therefore $A^{(0)} = cB^{(0)}/\omega$ follows from the fact that



energy is power divided by frequency. Equations (2.5.17) and (2.5.20) confirm that $B^{(0)}$ is proportional to *the quantum of power*, $\hbar\omega^2$,

$$B^{(0)} = \frac{\hbar\omega^2}{ec^2} = \frac{\hbar}{e}\kappa^2 . \tag{2.5.21}$$

The intensity equivalent to Eq. (2.5.21) gives *the radiation law for one photon*

$$I = \left( \frac{\hbar^2}{\mu_0 e^2 c^3} \right) \omega^4 . \tag{2.5.22}$$

This equation for one photon of energy $\hbar\omega$ is reminiscent of Stefan's law and Wien's law for black body radiation [21]; and it is deeply significant that these well known radiation laws stem from the fundamental relation between $B^{(0)}$ and $V$ (Eq. (2.5.5)). In the last analysis the radiation laws emanate from the existence of $V$, the volume occupied by a photon of energy $\hbar\omega$. The depth of insight provided by this relation is revealed by considering the density of states of classical electromagnetic oscillators, as given by the Rayleigh-Jeans law [21],

$$\frac{dN}{d\nu} = \frac{8\pi\nu^2}{c^3}, \tag{2.5.23}$$

where $N$ is the number of oscillators per $m^3$ and $\nu$ is the frequency $(\omega = 2\pi\nu)$. The density of states is therefore,

$$\frac{dN}{d\nu} = \frac{2}{\pi c}\kappa^2 = \frac{2e}{\pi c \hbar} B^{(0)}, \tag{2.5.24}$$

where we have used Eq. (2.5.21). In terms of the scalar, or Gaussian, curvature, $R$ [10] of the vacuum plane wave, we obtain,

$$\frac{dN}{d\nu} = \frac{2}{\pi c}R \, , \tag{2.5.25a}$$

$$B^{(0)} = \frac{\hbar}{e}R \, . \tag{2.5.25b}$$

In the generally relativistic theory of vacuum electromagnetism [10], $R$ is the Gaussian curvature of the Riemann tensor (Sec. 5.1), *showing that the Rayleigh-Jeans density of states, and the photomagneton* $\boldsymbol{B}^{(3)}$ *are both manifestations of space-time curvature,* $R = \kappa^2$. This inference allows radiation theory, notably the Planck distribution, to be developed as a theory of *general* relativity. At the most fundamental level, therefore, $\boldsymbol{B}^{(3)}$ is a property of curved space-time in general relativity, generated from the fact that the world-line of the charge quantum $e$ is the fiducial geodesic. The trajectory of $e$ in space is therefore a helix, and it becomes intuitively clear that this generates $\boldsymbol{B}^{(3)}$ along the axis of the helix (or *solenoid*).

The Planck distribution $\rho(\nu)$ is an expression for the mean energy $<\epsilon>$, of an electromagnetic oscillator of frequency $\nu$ when it can possess [21] only the discrete energies 0, $h\nu$, $2h\nu$, ...., $nh\nu$,

$$\begin{aligned}\rho(\nu) &= <\epsilon>\frac{dN}{d\nu} = \frac{2}{\pi c}R<\epsilon> = \frac{2e}{\pi c\hbar}B^{(0)}<\epsilon> \\ &= \frac{2}{\pi c}Rh\nu\left(\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\right) .\end{aligned} \tag{2.5.26}$$



### 5.3 Effect of Mass Density on Radiation Laws

The electromagnetic scalar curvature $R = \kappa^2$ appears therefore in the Planck distribution as a premultiplier. This is a scalar (or Gaussian) curvature in the theory of curvilinear coordinates [26], and Eq. (2.5.25b) demonstrates an *equivalence* between $R$ and the field component $B^{(0)}$. The scalar curvature $R_G$ from Einstein's equation [2] is, on the other hand,

$$R_G = -\frac{8\pi G}{c^2}\mu \, , \tag{2.5.27}$$

where $\mu$ is the mass density in kgm $m^{-3}$. Equation (2.5.27), in analogy with Eq. (2.5.25b), is an *equivalence* between $R_G$ and $\mu$, where $G$ is the gravitational constant [2],

$$G = 6.67 \times 10^{-11} \, m^3 \, \text{kgm}^{-1} \, s^{-2} \, . \tag{2.5.28}$$

Both $R$ and $R_G$ are geometrical scalar curvatures in the theory of curvilinear coordinates, with the same units ($m^{-2}$). It is therefore logical to assume that electromagnetic and gravitational curvatures are additive, i.e., that $R$ is changed to $R + R_G$ in the presence of mass density, $\mu$. If it is assumed that such an effect does *not* exist, i.e., that electromagnetic and gravitational fields do not mix in this way, then a major philosophical fault-line develops, in that there exists an equivalence principle in gravitation, but none in electromagnetism, and that in consequence, $R = 0$, there is no curvature in the space-time of electromagnetism. However, $R$ for a plane wave is $\kappa^2$, and is *not* zero. The received view treats electromagnetism [2] in a Galilean spacetime in which curvature is absent, but this clearly conflicts with $R = \kappa^2$; and recent work [4—10] has shown that it also conflicts with the existence of $\boldsymbol{B}^{(3)}$ because (Eq. (2.5.25b)) $\boldsymbol{B}^{(3)} = \Phi R \boldsymbol{e}^{(3)}$, where $\Phi = \hbar/e$

is the elementary fluxon (with the units of magnetic flux, weber) and where $\boldsymbol{e}^{(3)}$ is a unit vector in the propagation axis.

This section is therefore an attempt to develop simple cosmological tests for the hypothesis that $R$ and $R_G$ are additive. In direct logical consequence of this very simple ansatz, it can be shown as follows that the temperature and total photon density from a radiating black body are affected by its own mass density fluctuations. It may be possible to detect these small effects if the radiator is an object with a very large mass density $\mu$, perhaps a neutron star, or the as yet unobserved black hole. In its simplest form the calculation assumes that in the presence of mass density, $\mu$, the curvature of electromagnetism is changed by an amount determined from the Einstein equation,

$$\Delta R = (\Delta\kappa)^2 = \frac{8\pi G\Delta\mu}{c^2}, \tag{2.5.29}$$

so that the absolute change in electromagnetic frequency is proportional to the square root of the change in mass density,

$$\Delta\omega = (8\pi G)^{1/2}(\Delta\mu)^{1/2} \sim 4\times 10^{-5}(\Delta\mu)^{1/2}. \tag{2.5.30}$$

It is now assumed that this frequency correction due to mass density fluctuation is the same for all electromagnetic frequencies in a radiating black body. Mass density fluctuations in the radiator therefore affect its own radiation properties such as radiated intensity and radiated photons per unit volume at a detector. The change in radiated energy per unit volume and photon density due to a change in $\mu$ of the black body radiator (e.g. a dense cosmic source) are, respectively, with,

$$\Delta\nu_0 = \frac{\Delta\omega_0}{2\pi} = \left(\frac{2G}{\pi}\right)^{1/2}(\Delta\mu)^{1/2}, \tag{2.5.31a}$$



$$\Delta U = \int_0^{\Delta\nu_0}\frac{8\pi h}{c^3}\nu^3\left(\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\right)d\nu \sim \frac{8\pi kT}{3c^3}(\Delta\nu_0)^3, \tag{2.5.31b}$$

$$\Delta N = \int_0^{\Delta\nu_0}\frac{8\pi}{c^3}\nu^2\left(\frac{e^{-h\nu/kT}}{1-e^{-h\nu/kT}}\right)d\nu \sim \frac{2\pi kT}{hc^3}(\Delta\nu_0)^2, \tag{2.5.31c}$$

where we have used the classical approximation $h\nu/kT \ll 1$ [21]. The change in photon density for example is

$$\Delta N = \left(\frac{4kG}{hc^3}\right)T\Delta\mu \sim 10^{-25}\,T\Delta\mu \text{ photons } m^{-3}, \tag{2.5.32}$$

and is a small effect unless the product of $T$ and mass density fluctuations in the radiator are very large. In these calculations $k$ is Boltzmann's constant, and it is conjectured that there exists a physical upper bound on $\Delta\mu$; a physical mechanism which prevents the mass density of a radiating object from becoming infinite. Otherwise the electromagnetic frequency in Eq. (2.5.30) would also become infinite.

These small effects may be observable in solar physics with a very sensitive spectrometer with a sub-Hertzian resolution at visible frequencies. Although the solar mass is 1.989 x $10^{30}$ *kgm.*, the solar radius is 6.96 x $10^8$ m; and the mean mass density is of the order 1000 $kgm^{-3}$; i.e about a gram per *c.c.* This seems too small to see the effects proposed here, but in general the mass density depends on the gravitational scalar potential and orbital parameters [2]. Data from different cosmic objects with very large mass densities must probably be used to test our ansatz that electromagnetic curvature adds to gravitational curvature within unified field theory. The existence of $\boldsymbol{B}^{(3)}$ [4—10] is already an experimental indication that electromagnetic and gravitational fields are both geometrical in origin through a general principle of equivalence, and as we have seen, $\boldsymbol{B}^{(3)}$ is proportional directly to $R$.

**Acknowledgments**

York University, Canada and the Indian Statistical Institute are thanked for the award of visiting professorships. Many interesting Internet discussions are gratefully acknowledged.

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