# Circular and axial birefringence due to net angular momentum

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*Circular and axial birefringence due to net angular momentum is introduced in analogy with the Faraday and Wagnière/Meier effects. Spin induced circular birefringence conserves reversality and parity in both achiral and chiral ensembles, whereas the axial counterpart violates parity in achiral ensembles, and is supported in consequence only in a chiral molecular material. The new effects are treated with novel molecular property tensors in the nonrelativistic approximation, and distinguished carefully from the relativistic phenomenon of rotational aether drag.*

## Introduction

The theory of the electrodynamics of moving bodies implies translational aether drag, first described by J.J. Thompson[1], resulting in a tiny circular birefringence. This was later recognised as a relativistic phenomenon and was analysed by the young Fermi[2], whose result was correct but whose derivation, however, appears to be in error,[3] a mistake in the basic electrodynamics is cancelled, according to Player,[3] by the use of an incorrect form for rotationary power. We shall forgive young Fermi for this, in view of later contributions. R.V. Jones meticulously measured[4] the analogous relativistic phenomenon of rotational aether drag, which was analysed theoretically by Player.[3] Rotational aether drag is experienced by electromagnetic radiation propagating in Z through a medium (such as a long glass rod[4]) spinning about the same, Z, axis of the laboratory frame (X, Y, Z). The source of the radiation is in the same frame, and Jones demonstrated experimentally[4] the presence of a tiny circular birefringence whose origin is exclusively relativistic.

In this paper, circular[5] and axial[6-10] birefringence is introduced whose origin is NOT relativistic, which is caused by net macroscopic or microscopic angular momentum, and which is mediated by novel molecular property tensors in achiral and chiral molecular ensembles. This new nonrelativistic phenomenon should be distinguished from the outset from rotational aether drag.[3,4] Section 1 demonstrates that the effect conserves parity (P) and reversality (T) under appropriate circumstances; Section 2 introduces the mediating molecular property tensors; Section 3 provides expressions for the new effects.

## 1. Conservation of parity and reversality

The Wigner principles[11,12] of conservation of parity and reversality are applied in direct analogy to the Faraday and Wagnière Meier effects, because the reversality symmetry (T) and the parity inversion symmetry (P) of angular momentum (J) and magnetic flux density (B) are both the same. Both J and B are negative to T and positive to P. Electromagnetic radiation propagates with wave vector K in Z through a sample spinning about Z with angular momentum J. The T operator reverses both K and J, so that their relative directions are unchanged. T also leaves unchanged the circular and axial birefringence variables ($\theta_{\uparrow\uparrow} - \theta_{\downarrow\uparrow}$) and ($\eta_{\uparrow\uparrow} - \eta_{\downarrow\uparrow}$). The first of these is the difference between the refractive indices of the molecular ensemble measured with left and right circularly polarised radiation, in analogy with the Faraday effect.[12] The second, axial birefringence,[6-10] uses unpolarised radiation, and is the difference in refractive index measured with J parallel and antiparallel with K. T also leaves unchanged the coordinate system itself, an achiral molecular structure, represented, for example, by water, and a simple chiral molecular structure, represented, for example by an enantiomer of bromochloroflouromethane. The variables of the motion reversed complete experiment, as defined by Barron,[12] are therefore relatively all the same, and the experiment is therefore realisable.

Parity inversion, P, reverses K but leaves J unaffected, so that their relative directions are reversed. P leaves ($\theta_{\uparrow\uparrow} - \theta_{\downarrow\uparrow}$) the same but reverses ($\eta_{\uparrow\uparrow} - \eta_{\downarrow\uparrow}$). The coordinate system is reversed, but the achiral structure is indistinguishable after P and proper rotation.[12] P generates the opposite enantiomer of a chiral ensemble, and is therefore not a valid operation of the point group $R_h(3)$ of all rotations and reflections.[13-16] We conclude that circular birefringence due to J is possible in achiral and chiral ensembles, and reverses sign with J. Axial birefringence due to J is not possible in achiral ensembles,because P changes the sign of the variable (n - n) while leaving the achiral molecular structure the same. The P inverted variable and ensemble therefore bear an opposite relation to each other in the parity inverted experiment, and therefore violate the Wigner principle of parity conservation in the complete experiment.[12] Axial birefringence induced by J is possible, however, in CHIRAL ensembles, because P is not a valid symmetry operation in this case because it generates a different ensemble (the opposite enantiomer). The point group of a chiral ensemble (the opposite enantiomer). The point group of a chiral ensemble is the group of all rotations, $R(3)$,[13-16] not $R_h(3)$.

## 2. The mediating molecular property tensors

Section 1 has shown that circular and axial birefringence due to a sample spinning with a net angular momentum J conserves P and T in appropriate circumstances. The effect is not disallowed by the fundamental Wigner principles and merits further investigation. This is initiated in this section by assuming that the molecular polarisability and Rosenfeld tensors,[10,12] denoted respectively $\alpha_{2ij}$ and $\alpha_{1ij}$, may be expanded in a standard Voigt Born series in powers of J. Note that this is exactly analogous with the accepted Voigt Born expansion[12] of these molecular property tensors in powers of B. There is also an analogy between the interaction hamiltonian due to B and J. The former is

$$\Delta H = m \cdot B \tag{1}$$

where **m** is the molecular magnetic dipole moment, and the latter in a classically rotating sphere, for example, is

$$\Delta H = \frac{J^2}{2I^2} \equiv \left(\frac{J}{2I}\right) \cdot J \tag{2}$$

where I is the moment of inertia. The quantum equivalent of this expression is

$$(\Delta H)_{\text{quantum}} = \frac{\hbar^2}{2I} l(l+1)$$

where l is the rotational quantum number.

Considering the rotation of a molecule of an ensemble which is itself subjected to an external, macroscopic, angular momentum $J^{(1)}$, the total molecular anqular momentum becomes

$$J_{\text{tot}} = J + J^{(1)}$$

and the rotational hamiltonian becomes

$$\Delta H_1 = \left(\frac{J \cdot J + 2J \cdot J^{(1)} + J^{(1)} \cdot J^{(1)}}{2I}\right) \tag{3a}$$

The interaction part of this hamiltonian is therefore

$$\Delta H_2 = \frac{J}{I} \cdot J^{(1)} \tag{3b}$$

which consists of the dot product of the natural (or thermal) molecular angular momentum J, which exists in the absence of any macroscopically imposed angular momentum, and $J^{(1)}$, which is due to the externally applied net angular momentum. Note that the latter is a constant, independent of the properties and statistical dynamics of the molecular ensemble. The equivalent of "m" in the classical hamiltonian (2) is therefore "$-J/I$". Also, the T and P symmetries of **B** and $J^{(1)}$ are both the same (negative to T, positive to P) so that the P and T symmetries of the mediating, higher rank, Voigt Born tensors are the same for a perturbation caused either by **B** or by $J^{(1)}$. Again, in exact analogy with **B**, the perturbation caused by $J^{(1)}$ is a time odd influence, which 'activates' the time odd tensors of circular and axial optical activity.[12] In tensor notation:

$$\alpha_{1ij}\left(J^{(1)}\right) = \alpha_{1ij} + \alpha_{1ijk}^{(j)} J_k^{(1)} + \frac{1}{2}\alpha_{1ijkl}^{(jj)} J_k^{(1)} J_l^{(1)} + \dots \tag{4}$$

$$\alpha_{2ij}\left(J^{(1)}\right) = \alpha_{2ij} + \alpha_{2ijk}^{(j)} J_k^{(1)} + \frac{1}{2}\alpha_{2ijkl}^{(jj)} J_k^{(1)} J_l^{(1)} + \dots \tag{5}$$

where the effect of J on $\alpha_{1ij}$ and $\alpha_{2ij}$ is mediated by novel molecular property tensors of increasing rank. All molecular property tensors are complex in general[12,17] due to their quantum mechanical origins and properties. The overall T and P symmetries[12,17] on both sides of equations (4) and (5) must, however, be the same for the real and complex parts of the expansion. For our purposes it is sufficient to use the standard semiclassical Rayleigh or Maxwell theory[12] to solve for($\theta_{\uparrow\uparrow} - \theta_{\downarrow\uparrow}$) and ($\eta_{\uparrow\uparrow} - \eta_{\downarrow\uparrow}$)due to $J^{(1)}$, the Taylor expansions are truncated at

first order in $J^{(1)}$, and the property tensors written out in terms of real (single primed) and complex (double primed) components

$$\alpha_{1ij} = \alpha'_{1ij} - i\alpha''_{1ij} \tag{6}$$

$$\alpha_{2ij} = \alpha'_{2ij} - i\alpha''_{2ij} \tag{7}$$

$$\alpha^{(j)}_{1ijk} = \alpha'^{(j)}_{1ijk} - i\alpha''^{(j)}_{1ijk} \tag{8}$$

and so on.

In analogy with the theory of axial and circular birefringence due to **B**, we shall also require the Voigt Born expansion of the electric dipole/electric quadrupole molecular property tensor[12] $A^{ijk}$, as follows

$$A_{ijk}\left(J^{(1)}\right) = A_{ijk} + A^{(j)}_{ijkl}J^{(1)}_l + \ldots \tag{9}$$

Note that the mediating tensors in all of these Voigt Born expansions are well defined quantum mechanically. For example, the tensor $\alpha_{2ijk}$ can be defined by an expression similar to that in equation (4.9a) of Barron and Vrbancich.[9] Specific consideration of this is given in the Appendix.

## 3. The circular and axial birefringence

Expressions for axial and circular birefringence from the above Voigt Born expansions can now be deduced by a straightforward adaptation of the standard theory of the Faraday and Wagnière Meier effects[9,12]. This uses the zeta tensors

$$\zeta_{ijk} = \zeta'_{ijk} - i\zeta''_{ijk} \tag{10}$$

In the presence of a time-odd influence, such as $J^{(1)}$, all components of the zeta tensors contribute[12] to axial birefringence, due specifically to $J^{(1)}$

### a) Circular birefringence

This is the difference between the real refractive index of the spinning sample measured with left (L) and right (R) circularly polarised radiation. The corresponding difference in the imaginary parts is the circular dichroism. From the Rayleigh scattering model of refraction, as developed by Barron, [12] and also from the Maxwell equations, the real parts of the refractive index of a sample as measured by right and left circularly polarised radiation are, respectively

$$n'_R = 1 + \frac{1}{4}\mu_0 c^2 N\left[\alpha'_{1XX}(f) + \alpha'_{1YY}(f)\right. \tag{11}$$

$$\left. +\zeta''_{XXZ}(f) + \zeta''_{YYZ}(f) - 2\left(\alpha''_{1XY}(f) + \zeta''_{XYZ}(f)\right) + \cdots \right]$$

$$n'_L = 1 + \frac{1}{4}\mu_0 c^2 N\left[\alpha'_{1XX}(f) + \alpha'_{1YY}(f)\right. \tag{12}$$

$$\left. +\zeta''_{XXZ}(f) + \zeta''_{YYZ}(f) + 2\left(\alpha''_{1XY}(f) + \zeta''_{XYZ}(f)\right) + \cdots \right]$$

The corresponding imaginary parts are

$$n_R'' = \frac{1}{4}\mu_0 c^2 N\left[\alpha_{1XX}'(g) + \alpha_{1YY}'(g)\right.$$

$$\left. + \zeta_{xxz}'(g) + \zeta_{yyz}''(g) - \left(2\alpha_{XY}''(g) + \zeta_{XYZ}'''(g)\right) + \cdots\right] \tag{13}$$

$$n_L'' = \frac{1}{4}\mu_0 c^2 N\left[\alpha_{1XX}'(g) + \alpha_{1YY}'(g)\right.$$

$$\left. + \zeta_{xxz}'(g) + \zeta_{yyz}''(g) + 2\left(\alpha_{1XY}''(g) + \zeta_{XYZ}'''(g)\right) + \cdots\right] \tag{14}$$

Circular birefringence is defined as the difference

$$(n_L' - n_R') = \left(\alpha_{1XY}'''(f) + \zeta_{XYZ}'''(f)\right)\mu_0 c^2 N \tag{15}$$

and circular dichroism as

$$(n_L'' - n_R'') = \left(\alpha_{1XY}''(g) + \zeta_{XYZ}''(g)\right)\mu_0 c^2 N \tag{16}$$

Here $\mu_0$ is the permeability in *vacuo*, N the number of molecules per cubic metre, and c the velocity of light. In these expressions, the molecular property tensors have been split[12] into dispersive (f) and absorptive (g) components. The polarisibility, for example, becomes

$$\alpha_{1ij}' \rightarrow \alpha_{1ij}'(f) + i\alpha_{1ij}'(g) \tag{17}$$

with the quantum definitions[12]

$$\alpha_{1ij}'(f) = \frac{2}{\hbar}f\omega_{\alpha n}\text{Re}\left(\langle n|\mu_i|\alpha\rangle\langle\alpha|\mu_j|n\rangle\right) \tag{18}$$

$$\alpha_{1ij}'(g) = \frac{2}{\hbar}g\omega_{\alpha n}\text{Re}\left(\langle n|\mu_i|\alpha\rangle\langle\alpha|\mu_j|n\rangle\right) \tag{19}$$

The circular dichroism in the power absorption coefficient (neper $cm^{-1}$) is

$$A_z^{circ} = 2\frac{\omega}{c}\left(n_L'' - n_R''\right) \tag{20}$$

and the angle of rotation of plane polarised radiation is

$$\Theta = \frac{1}{2}l\frac{\omega}{c}\left(n_L' - n_R'\right) \tag{21}$$

where l is the sample length.

## b) Axial birefringence

In this case the probe is unpolarised[6-10], not plane polarised as in circular birefringence. It measures an average refractive index

$$n_{av}' = \frac{1}{2}\left(n_L' + n_R'\right) \tag{22}$$

and axial birefringence is the difference in this AVERAGE with J parallel and antiparallel to the light beam in Z. From the theory of axial birefringence due to Barron and Vrbancich[9] we have the following expressions for refractive and absorption indices in unpolarised light

$$n'_{av} = 1 + \frac{1}{4}\mu_0 c^2 N \left[ \alpha'_{1XX}(f) + \alpha'_{1YY}(f) \right.$$

$$\left. + \zeta''_{xxz}(f) + \zeta''_{yyz}(f) \right]$$  (23)

and
$$n''_{av} = \frac{1}{4}\mu_0 c^2 N \left[ \alpha'_{1XX}(g) + \alpha'_{1YY}(g) \right.$$

$$\left. + \zeta''_{xxz}(g) + \zeta''_{yyz}(g) \right]$$  (24)

which can be related to the real and imaginary parts of the complex permittivity[18,19] by

$$\varepsilon' = n'^2_{av} - n''^2_{av}; \varepsilon'' = 2n''_{av}n'_{av}$$  (25)

## c) Circular and axial birefringence induced by $J^{(1)}$

The circular birefringence is obtained from equation (15), and ensemble averaged.[9,12] In direct analogy with the semiclassical theory of the Faraday effect (circular birefringence and dichroism due to $\mathbf{B}$) we have nonrelativistic circular birefringence and dichroism due to net angular momentum, $J^{(1)}$ given by

$$(n'_L - n'_R) = \mu_0 c^2 N \alpha''_{1xy}(f) + \ldots$$  (26)

where the time-odd tensor component $\alpha_{1xy}{}''(f)$ is activated through a Voigt Born perturbation to first order in $\mathbf{J}$, a time odd influence:

$$\alpha''_{1xy}(f, J^{(1)}) = \alpha''_{1xy}(f) + \alpha''^{(1)}_{1xy} J^{(1)}_z + \ldots$$  (27)

and ensemble averaged using an interaction energy due to $\mathbf{J}$. In a rod rotating about Z with angular momentum $J_z$, such as that used by R.V. Jones,[4] this can be expressed from equation (3a) as

$$V = \left( \frac{J_z}{I} \right) J^{(1)}_z$$  (28)

This leads to the final ensemble averaged expression for circular birefringence due to net $J_z^{(1)}$, with

$$\langle \alpha''_{1xy} \rangle = \left( \alpha''^{(1)}_{1ijk} - \frac{J_z}{IkT} \alpha''_{1ij} \right) \langle i_i j_j k_k \rangle J^{(1)}_z$$  (29)

It is directly proportional to $J_z^{(1)}$, and reverses sign with $J_z^{(1)}$. The angle of rotation is

$$\Theta = \frac{1}{12}\omega\mu_0 c N J^{(1)}_z \varepsilon_{ijk} \left( \alpha''^{(1)}_{1ijk}(f) - \frac{J_z}{I} \frac{\alpha''_{1ijk}(f)}{kT} \right) + \ldots$$  (30)

The effect exists in all molecular ensembles, in analogy with the Faraday effect. The terms analogous to the Faraday A, B, and C terms are obtained from equation (30) as in the standard theory of the Faraday effect.[12] An order of magnitude estimate of the effect can be made from equation (30), using the relation

$$J^{(1)}_z = I\Omega^{(1)}_z$$  (31)

where $\Omega_z^{(1)}$ is the angular velocity of the spinning sample, for example a rod as used by R.V. Jones.[4] In equation (30) we have $kT = 414 \times 10^{-23}$ J at 300K; I, the molecular moment of inertia, is taken typically as $500 \times 10^{-47}$ kg m$^2$. Using $N = 6 \times 10^{26}$ m$^{-3}$; $\mu_0 = 4\pi \times 10^{-7}$ J m$^{-1}$ A$^{-2}$; $c = 3 \times 10^8$ m s$^{-1}$; a net angular velocity of $10^6$ radians s$^{-1}$; an order of magnitude of $10^{-38}$ J$^{-1}$ C$^2$ m$^2$ for $\alpha_{1ijk}''$, and visible sodium D line radiation at about $\omega = 10^{16}$ radians per second, we obtain

$$\Theta = 10^{-6} / \text{radians} \tag{32}$$

so for a rod a metre in length, the angle of rotation of plane polarised radiation sent down the Z (rotational) axis of the rod would be the order of magnitude of $10^{-6}$ radians. This is in range of the apparatus constructed by R.V. Jones[4] for rotational aether drag, a purely relativistic effect.

The effect is proportional to the square of the net rotational velocity, to the molecular moment of inertia I, and to the molecular polarisability elements $\alpha_{1ijk}''(f)$ in the molecule fixed frame of reference. The angular momentum $J_z^{(1)}$ can be generated mechanically, or in a gaseous or liquid sample, by using rotating electric fields, a method first suggested by Born[20] and demonstrated by Lertes,[21] and Grossetti.[22] It has been discussed in depth by Dahler.[23] Rotating electric fields can probably provide $\Omega_z$ in the range up to $10^8$ radians per second, and the sample can be kept stationary. In addition there will be dispersive effects on the observable angle of rotation in analogy with dielectric spectroscopy,[24] *i. e.* at very high spinning electric field frequencies the molecules of the sample are not able to acquire the full angular velocity of the rotating electric field. This was realised by Born[20] shortly after the appearance of Debye's theory of dielectric loss at radio frequencies.[25] The axial birefringence from equation (23) is

$$\left(n'_{\uparrow\uparrow} - n'_{\uparrow\downarrow}\right) = \mu_0 c N J_z^{(1)} \left[ \frac{\omega}{45} \left(3A_{i\mu}^{\prime\prime(1)}(f)\right) \right.$$

$$-A_{\mu}^{\prime\prime(1)}(f) - \left(3A_{\bar{\mu}\bar{\jmath}}''(f)\frac{J_z}{I} - A_{\bar{\mu}}''(f)\frac{J_z}{I}\right)\Big/kT\right) \tag{33}$$

$$\left. +\frac{1}{3}\varepsilon_{ijk}\left(\alpha_{2ijk}^{(1)}(f) - \frac{J_z}{I}\alpha_{2ij}(f)\right)\Big/kT\right) + ... \right]$$

in analogy with equation (3.17a) of Barron and Vrbancich for the magnetochiral effect.[9] This effect can occur only in chiral ensembles, because it would otherwise violate parity reversal symmetry. The averaged axial birefringence due to net angular momentum in a chiral sample is proportional to the square of net angular velocity, and the effect is much smaller, probably requiring for observation a method of inducing NET angular momentum at infra red or visible frequencies in a molecular vapour, *i.e.* angular momentum that survives ensemble averaging.

### Discussion

There appears to be only one brief mention in the literature, that by Barron,[26] of the possibility of the effects described in equations (30) to (33). Barron anticipated that circular birefringence as in equations (30) to (32) would be a 'magnetic' type of optical activity, as opposed to 'natural'. He appears to have made no analysis of the axial equivalent, described in equation (33). It is clear from the third principle of group theoretical statistical mechanics,[14-16] however, that the symmetry of J is imparted to new ensemble averages with the same symmetry, and this is seen clearly in equations (13) to (16). In achiral ensembles

the symmetry of the effect is $D_g^{(I)}$, and in chiral ensembles it is $D^{(I)}$.

Clearly, it may be generated by ultracentrifuging, in the manner of R.V. Jones,[4] a molecular crystal or liquid with windows for a spectropolarimeter to measure the circular birefringence. Plotting $\Theta$ vs. $J_z^{(1)}$ of the ultracentrifuge gives the scalar $\alpha_{1ijk}''^{(J)}(f)$ and a new fundamental information about the molecule. An extension of this investigation gives spectral information, such as circular dichroism and optical rotatory dispersion induced by $J_z^{(1)}$. The corresponding axial effects are much more difficult to see, being orders of magnitude smaller, but if observed, again give new fundamental molecular constants.

It appears equally clear that similar effects would be observable by spinning the source and keeping the sample steady. There appear to be several consequences of the effect in astrospectroscopy, simply because the earth spins diurnally and also orbits the sun. These are distinct from the purely relativistic phenomenon of light aberration.[27] For example, the orbiting Hubble space telescope carrying a spectropolarimeter would see circular birefringence in an achiral sample with source radiation from the sun, birefringence induced by the earth's spin.

Optical activity induced by $J^{(1)}$ is also produced, in principle, by net microscopic angular momentum which survives ensemble averaging. An example has been given by Baranova and Zeldovich,[28] using a radio frequency field. Another possibility, as mentioned, is the Born effect,[20] where a liquid sample suspended on a torsion wire attains a net $J^{(1)}$ through a spinning electric field. The sample develops optical activity through the equations of this paper. Another example is the Beth effect,[29] where a chiral crystal suspended on a torsion wire is rotated by circularly polarised pump radiation. In general, spin induced optical activity will appear in any experimental configuration where there is a net excess of angular momentum which survives ensemble averaging, for example if rotational quantum K states are unequally populated as the result of some external perturbation. A possible source of such perturbation is a spin polarised electron or neutron beam, inducing spin optical activity measurable by a probe laser.

## Conclusion

Circular and axial birefringence and dichroism has been demonstrated due to net angular momentum $J^{(1)}$, and the circumstances defined under which it conserves parity and reversality.

## Acknowledgements

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## Appendix

In this appendix we make the first inroads to what appears to be a profound 'new look' at the role of angular momentum in physics.[30] The remarks in this appendix are confined to the role of net angular momentum in standard quantum perturbation theory. This treatment follows that of Barron and Vrbancich in reference 9, p.723, section 4. Quantum mechanical expressions for the perturbed tensors $\alpha_{1ijk}{}^{(J)}$ and $\alpha_{2ijk}{}^{(J)}$ of the text of this paper may be found using wavefunctions and energies perturbed by net angular momentum, which for the sake of simplicity is kept classical (equation (2)). The eigenfunction $\psi_j'$ and energy eigenvalue $W_j'$ perturbed to first order in the energy are:

$$\psi_j' = \psi_j + \frac{J_{Z_Y}^{(l)}}{\hbar} \sum_{k \ne j} \frac{1}{\omega_{jk}} \left\langle k \left| \frac{J_{Z_Y}}{I} \right| j \right\rangle \psi_k \tag{A1}$$

$$W_j' = W_j + \left\langle j \left| \frac{J_{Z_Y}}{I} \right| j \right\rangle J_{Z_Y}^{(l)} \tag{A2}$$

which, as explained in reference 9, are valid even if the unperturbed

eigenfunction $\psi_j$ belongs to a degenerate set, provided the degenerate eigenfunctions are chosen to be diagonal in the perturbation. The frequency separation of the levels perturbed by (classical) net angular momentum is

$$\omega'_{jn} = \omega_{jn} - \left(\frac{J_{Z_{jj\gamma}}}{I} - \frac{J_{Z_{nn\gamma}}}{I}\right)\frac{J_{Z\gamma}^{(l)}}{\hbar} \tag{A3}$$

where

$$J_{Z_{jj\gamma}} = \left\langle j\left|\frac{J_{Z\gamma}}{I}\right|j\right\rangle \tag{A4}$$

is for the molecule in the perturbed state $\psi_j$. Expressions for the perturbing tensors then follow straightforwardly. For example

$$\begin{aligned}
\alpha_{2\alpha\beta\gamma}^{(l)} &= \frac{2}{\hbar^2}\sum_{j\neq n}\left\{\frac{\omega_{jn}^2+\omega^2}{\left(\omega_{jn}^2-\omega^2\right)^2}\left(\frac{J_{Z_{jj\gamma}}}{I}-\frac{J_{Z_{nn\gamma}}}{I}\right)\right. \\
&\quad \times \mathrm{Re}\left(\langle n|\mu_\alpha|j\rangle\langle j|m_\beta|n\rangle\right) \\
&\quad +\sum_{k\neq n}\frac{\omega_{jn}}{\omega_{kn}\left(\omega_{jn}^2-\omega^2\right)}\mathrm{Re}\left[\left\langle k\left|\frac{J_{Z\gamma}}{I}\right|n\right\rangle\left(\langle n|\mu_\alpha|j\rangle\langle j|m_\beta|k\rangle\right.\right. \\
&\quad \left.+\langle n|m_\beta|j\rangle\langle j|\mu_\alpha|k\rangle\right)\right]+\sum_{k\neq j}\frac{\omega_{jn}}{\omega_{kj}\left(\omega_{jn}^2-\omega^2\right)} \\
&\quad \times \mathrm{Re}\left[\left\langle j\left|\frac{J_{Z\gamma}}{I}\right|k\right\rangle\left(\langle n|\mu_\alpha|j\rangle\langle k|m_\beta|n\rangle\right.\right. \\
&\quad \left.\left.\left.+\langle n|m_\beta|j\rangle\langle j|\mu_\alpha|n\rangle\right)\right]\right\}
\end{aligned} \tag{A5}$$

This shows that net angular momentum plays the same role in spectroscopy as a magnetic field, i.e., lifts degeneracy. In this paper we have confined discussion to the nonrelativistic approximation, but a fuller relativistic treatment would introduce many more new effects involving the rotational aether drag[1-4] interacting with the new concepts introduced here. One particularly interesting example of the fundamental role of angular momentum in physics and relativistic cosmology comes from the Dirac equation. This produces a doubly degenerate solution at low kinetic energies in the absence of a perturbation, which is usually taken as a magnetic field. The results of this paper show that this perturbation COULD EQUALLY WELL BE NET ANGULAR MOMENTUM. This removes the degeneracy and leads to the entirely new concept that electron spin, with quantum number s = 1/2, may be induced by net angular momentum. The equivalent of the spin 'magnetic moment' is, in the classical limit, J/(I). Now this quantity may be generated BY THE MOLECULE ITSELF, provided it has net angular momentum, so the famous Uhlenbeck/Goudsmit hypothesis emerges without further ado. This is one example out of many consequences of the role of net angular momentum in physics.