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THE PHOTON'S MAGNETOSTATIC FLUX QUANTUM:

SYMMETRY AND WAVE PARTICLE DUALITY, FUNDAMENTAL

CONSEQUENCES IN PHYSICAL OPTICS.

by

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ABSTRACT

The recent discovery of the photon's magnetostatic flux quantum ($\hat{B}_{\pi}$) is explored in new fundamental directions. It is shown that there exists a novel symmetry duality in Maxwell's equations in free space which shows that the classical equivalent of $\hat{B}_{\pi}$, the axial vector $\mathbf{B}_{\pi}$, can be defined simultaneously in terms of scalar and pseudoscalar magnitudes $(B_0)_+$ and $(B_0)_-$, respectively. In the quantised field this is interpreted as a duality in the photon's angular and linear momentum operators $\hat{J}$ and $\hat{p}$ respectively, each of which can be used to define $\hat{B}_{\pi}$. This is shown to be a generalization for the photon of the fundamentally important de Broglie wave particle duality. Some consequences for elementary particle theory are discussed qualitatively. It is argued that the operator $\hat{B}_{\pi}$, or its classical equivalent, $\mathbf{B}_{\pi}$, is responsible for other fundamental phenomena of physical optics, among which are ellipticity in an electromagnetic plane wave, ellipticity developed in the measuring beam of the Kerr effect, and circular dichroism, each of whose ~~whose~~ origins are therefore shown for the first time to be magneto-optic and fundamentally dependent upon $\hat{B}_{\pi}$. In general, the ubiquitous, pseudoscalar, Stokes third parameter S_3 is shown to be directly proportional to the pseudoscalar magnitude $(B_0)_-$, and the quantised third Stokes operator, $\hat{S}_3$ recently introduced by Tanaś and Kielich (1) is proportional to the operator $\hat{B}_{\pi}$.

1. INTRODUCTION.

It has recently been demonstrated theoretically that there exists an operator $\hat{B}_\pi$ of the quantised electromagnetic field that describes the photon's magnetostatic flux density:

$$\hat{B}_\pi = B_0 \frac{\hat{J}}{\hbar} . \qquad (1)$$

Here B_0 has been interpreted (2-5) as a scalar magnetic flux density amplitude of a beam of circularly polarised light consisting of one photon, and $\hat{J}$ is the well known boson operator (6,7) describing that photon's quantised angular momentum. The classical equivalent of $\hat{B}_\pi$ is a novel axial vector $\underset{\sim}{B}_\pi$, which is directed in the propagation axis of the beam. In this paper it is demonstrated using elementary tensor algebra, and from inspection of the Maxwell equations of the classical field, that there is another possible interpretation of the scalar amplitude B_0, designated henceforth by $(B_0)_+$, where the + subscript is to be interpreted as "positive to parity inversion". It turns out that B_0 can be interpreted both as a scalar and as a pseudoscalar quantity designated $(B_0)_-$, where the subscript - means "negative to parity inversion". This is designated "symmetry duality", and is shown in this work to imply that $\hat{B}_\pi$ can be defined simultaneously in terms of the photon's angular momentum operator $\hat{J}$ and linear momentum operator $\hat{p}$, a result which is a generalisation of a keystone of wave mechanics, the de Broglie wave particle duality (6). The latter is linked through $\hat{B}_\pi$ to a symmetry duality in Maxwell's classical equations.

It has already been shown theoretically (2-5) and experimentally (8, 9) that circularly polarised light can magnetize, leading for example to the inverse Faraday effect (10-13) and novel, potentially very useful, light induced shifts in NMR spectroscopy (8, 9) in one and more dimensions. The existence of the operator $\hat{B}_{\pi}$, and its classical equivalent $\underset{\sim}{B}_{\pi}$, makes it much easier to interpret these magnetization effects by treating circularly polarised light as a "magnet" generating this novel flux quantum per photon. The $\hat{B}_{\pi}$ concept also makes it relatively straightforward to forecast the existence of novel spectral phenomena such as optical Zeeman, anomalous Zeeman and Paschen Back effects (3), an optical Faraday effect and optically induced magnetic circular dichroism (4), an optical Stern Gerlach effect, using a focused laser beam to produce a light induced magnetic field gradient, optical ESR effects, optically induced effects in interacting beams, such as a beam of circularly polarised photons reflected (5) from a beam of polarised electrons, and so on. All these effects can be thought of as arising from the replacement (or augmentation) of an ordinary magnet by or with a circularly polarised laser. These theories allow scope for the development of several novel analytically useful methods.

In this paper it is shown that $\underset{\sim}{B}_{\pi}$ is related directly to the ubiquitous (14), pseudoscalar, third Stokes parameter S_3 of the classical electromagnetic plane wave, which becomes in quantum field theory the third Stokes operator of Tanaś and Kielich (1). Therefore it follows immediately that several well known phenomena of physical optics can be re-interpreted fundamentally in terms of the operator $\hat{B}_{\pi}$, or its classical equivalent $\underset{\sim}{B}_{\pi}$. Examples include ellipticity in the plane wave, ellipticity developed in the measuring beam of the electrical Kerr effect, and circular dichroism, which are shown in this work to be magneto-optic phenomena. Therefore, not only does $\hat{B}_{\pi}$ allow this re-interpetation, in both classical and quantum field theory, but it also allows a link to be made between de Broglie wave particle duality and symmetry duality in the classical Maxwell equations. It appears, therefore, to go to the root of physical optics and field theory.

In Section 2 the mathematical basis of symmetry duality is developed with elementary vector and tensor algebra, before embarking in Section 3 on a discussion of symmetry duality in the link between $\underset{\sim}{B}_{\pi}$ and S_3 . Section 4 develops the link between wave particle duality and the symmetry duality in Maxwell's equations dmonstrated in Section 3, and discusses qualitatively the implications for elementary particle theory. Section 5 develops the link between $\underset{\sim}{B}_{\pi}$ and S_3 into a novel explanation for ellipticity and circular dichroism in physical optics.

2. SYMMETRY DUALITY IN THE VECTOR PRODUCT OF TWO POLAR VECTORS.

It is well known that the components of a vector which can be written as the cross product of two polar vectors do not change sign under parity inversion ($\hat{P}$) and that the vector so formed is an axial vector (15), or pseudovector. The conjugate product of the classical electromagnetic field [2-5]:

$$\underset{\sim}{\Pi}^{(A)} = \underset{\sim}{E} \times \underset{\sim}{E}^{*} = 2\left(E_0^2\right)_+ i\, \underset{\sim}{e}_+ , \quad \text{— (2)}$$

where $\underset{\sim}{E}^{*}$ is the complex conjugate of the electric field strength vector $\underset{\sim}{E}$, is an axial vector, therefore. Here, $\underset{\sim}{e}_+$ is an axial unit vector, positive to $\hat{P}$, and the quantity $\left(E_0^2\right)_+$ is a scalar, also positive to $\hat{P}$. The overall motion reversal ($\hat{T}$) symmetry of $\underset{\sim}{\Pi}^{(A)}$ is negative, and it is natural to define $\underset{\sim}{e}_+$ as a $\hat{T}$ negative unit vector; so that $\left(E_0^2\right)_+$ is a $\hat{T}$ positive scalar.

It appears at first sight that these definitions are both necessary and sufficient for the complete definition of the axial vector $\underset{\sim}{\Pi}^{(A)}$; but mathematically, there is an alternative, which is revealed through writing any arbitrary axial vector as

$$\underset{\sim}{C} = C_+ \underset{\sim}{e}_+ = C_- \underset{\sim}{e}_- \quad \text{— (3)}$$

where C_+ and $\underset{\sim}{e}_+$ are respectively $\hat{P}$ positive scalar and unit axial vector quantities, and where C_- and $\underset{\sim}{e}_-$ are respectively $\hat{P}$ negative pseudoscalar and $\hat{P}$ negative polar unit vector quantities. The overall $\hat{P}$ symmetry of the complete axial vector $\underset{\sim}{C}$ is positive in both cases.

This seemingly mundane observation in elementary vector analysis has far reaching consequences in the theory of the classical and quantised electromagnetic fields.

In tensor algebra, the general vector cross product $\underset{\sim}{C} = \underset{\sim}{A} \times \underset{\sim}{B}$ is written, as is well known, with the third rank antisymmetric (or alternating) unit tensor, $\epsilon_{\alpha\beta\gamma}$, known as the Levi Civita symbol (7, 15):

$$C_\alpha = \frac{1}{2}\epsilon_{\alpha\beta\gamma}\left(A_\beta B_\gamma - A_\gamma B_\beta\right) \equiv \frac{1}{2}\epsilon_{\alpha\beta\gamma} C_{\beta\gamma} \quad -(4)$$

where the $\hat{P}$ symmetry of $\epsilon_{\alpha\beta\gamma}$ is negative, so that $C_{\beta\gamma}$ is a $\hat{P}$ negative antisymmetric polar tensor of rank two. Evidently, C_α must be $\hat{P}$ positive, and is the rank one axial tensor (i.e. an axial vector). However, $C_{\beta\gamma}$ can also be written (15) as

$$C_{\beta\gamma} = -i\,\epsilon_{\beta\gamma} C_- \quad —(5)$$

where $\epsilon_{\beta\gamma}$ is the $\hat{P}$ positive, axial, unit antisymmetric tensor of rank two, and C_- is the pseudoscalar of eqn. (3). Eqn (5) shows that the polar antisymmetric tensor of rank two can be reduced, quite generally, to a pseudoscalar, a particular result of a generalization well known (15) in the relativistic theory of the classical electromagnetic field. Note that $C_{\beta\gamma}$ is purely imaginary from the hermitian properties of the general second rank tensor, which can always be written as a sum of real symmetric and imaginary antisymmetric parts (15).

Therefore

$$C_\alpha = -\frac{i}{2}\epsilon_{\alpha\beta\gamma}\epsilon_{\beta\gamma}C_- \qquad -(6)$$

or

$$C_\alpha \equiv -iC_+\epsilon_{\alpha+} = -\frac{i}{2}\epsilon_{\alpha\beta\gamma}\epsilon_{\beta\gamma}C_- \qquad -(7)$$

where $\epsilon_{\alpha+}$ is the rank one axial unit tensor, positive to $\hat{P}$, and C_+ is a $\hat{P}$ positive scalar. Recall that C_- is a $\hat{P}$ negative pseudoscalar. Eqns. (3) and (7), using vector and tensor notation respectively, are expressions of that which we denote "symmetry duality", a purely mathematical result which shows that a scalar and pseudoscalar may both be used to define an axial vector. Clearly, if we take the magnitude ($|\underline{C}|$) of the axial vector $\underline{C}$ in eqn (3) we obtain

$$\left.\begin{aligned} C^2 &\equiv \underset{\sim}{C}\cdot\underset{\sim}{C} = C_+^2 = C_-^2 \quad ; \\ |\underset{\sim}{C}| &= |(C^2)^{1/2}| = |C_+| = |C_-| ; \end{aligned}\right\} -(8)$$

so that the positive parts of the scalar C_+ and pseudoscalar C_- are equal in absolute magnitude. This same result can be obtained from the tensor eqn. (7) by taking a particular Z component:

$$C_Z \equiv -iC_+\epsilon_{Z+} = -\frac{i}{2}C_-\left(\epsilon_{ZXY}\epsilon_{XY} + \epsilon_{ZYX}\epsilon_{YX}\right) \qquad -(9)$$

where the well known Einstein convention of summation over repeated indices has been used on the right hand side. With the component definitions: $\epsilon_{ZXY} = 1$; $\epsilon_{XY} = 1$; $\epsilon_{ZYX} = -1$; $\epsilon_{YX} = -1$; we obtain

$$C_{+} \epsilon_{Z+} = C_{-} \epsilon_{Z-} \qquad -(10)$$

where

$$\epsilon_{Z-} \equiv \epsilon_{ZXY} \epsilon_{XY} + \epsilon_{ZYX} \epsilon_{YX} \qquad -(11)$$

is the Z component of the $\hat{P}$ negative polar unit tensor of rank one, $\epsilon_{\alpha -}$. Note that eqns. (3) and (10) are identical in symmetry for the considered Z components of $\underline{C}$. Eqn. (10), which is a direct and fundamental consequence of elementary tensor algebra, again shows the symmetry duality between scalar and pseudoscalar in the definition of the axial, or pseudo, vector.

It is now possible to apply the purely mathematical principle of symmetry duality to the classical, non-relativistic (or relativistic), field to obtain novel information of fundamental importance in physical optics, particularly in respect of a $\hat{P}$ positive, $\hat{T}$ negative, axial vector, a novel magnetostatic field, $\underline{B}_{\pi}$, (2 - 5) associated with the electromagnetic plane wave, or in the quantised field, the magnetostatic flux density operator, $\hat{B}_{\pi}$, of the photon.

3. AN EXAMPLE OF SYMMETRY DUALITY, THE RELATION BETWEEN $\underset{\sim}{B}_\pi$ AND THE STOKES PARAMETER S_3 .

Consider the classical electromagnetic wave in free space, so that the real scalar refractive index is unity. It follows from Maxwell's equations for a plane wave that

$$E_0 = c B_0 \qquad \text{— (12)}$$

where E_0 and B_0 are $\hat{P}$ and $\hat{T}$ positive scalars, amplitudes, respectively, of the electric field strength and magnetic flux density. The intensity of the wave is defined in free space by

$$I_0 = \epsilon_0 c E_0^2 \qquad \text{— (13)}$$

where ϵ_0 is the free space permittivity (6) in S.I. units, and c the speed of light in vacuo. With eqns (12) and (13), eqn (2) can be rewritten as

$$\underset{\sim}{\Pi}^{(A)} = 2(E_0)_+ c i \underset{\sim}{B}_\pi \qquad \text{— (14)}$$

where we have defined the **magnetostatic flux density vector** $\underset{\sim}{B}_\pi$ [2-5] of the classical electromagnetic plane wave in free space:

$$\underset{\sim}{B}_\pi \equiv (B_0)_+ \underset{\sim}{e}_+ \qquad \text{— (15)}$$

where $\underset{\sim}{e}_+$ is a $\hat{P}$ positive unit axial vector. The overall $\hat{T}$ symmetry of $\underset{\sim}{B}_\pi$ is negative, and its overall $\hat{P}$ symmetry is positive. In the introduction we have given an account of the role of $\underset{\sim}{B}_{\pi}$ in the re-interpretation of well known effects, such as circular dichroism, and ellipticity, and its mediating role in new effects such as optical NMR and ESR (9), optical Faraday (4) and Zeeman (3) effects, optical Stern Gerlach effects, optical Compton scattering (5) and so on (2-5). Its quantised equivalent is the magnetostatic flux density operator of eqn. (1), in which $(B_0)_+$ is defined as the $\hat{P}$ and $\hat{T}$ positive scalar magnetic flux density amplitude of one photon.

Again, as in Section 2, it would appear at first sight as if the definition of the seemingly mundane quantity B_0 as a $\hat{P}$ and $\hat{T}$ positive scalar is sufficient. Remarkably, however, this is not the case, there is an alternative definition possible of the novel classical vector $\underset{\sim}{B}_{\pi}$ which uses B_0 as a **pseudoscalar** and not only does this emerge naturally from the Maxwell equations for the plane wave, but also provides a natural link between $\underset{\sim}{B}_{\pi}$ and the well known third Stokes parameter S_3 (1, 7, 14, 15).

These conclusions emerge straightforwardly from the well known equations linking the $\underset{\sim}{E}$ and $\underset{\sim}{B}$ vectors of the classical electromagnetic plane wave in a medium of refractive index n, defined through the well known classical wave vector, $\underset{\sim}{\kappa}$, a $\hat{T}$ and $\hat{P}$ negative polar vector directed in the propagation axis, Z, of the plane wave (7):

$\underset{\sim}{\kappa}$ l.c. bold kappa.

$$\underset{\sim}{\kappa} = \frac{\omega}{c}\,\underset{\sim}{n}\ ; \quad n = \frac{c}{v}. \qquad (16)$$

Here ω is the angular frequency in radians per second of the plane wave, as usual. Maxwell's equations give the well known (7):

$$\underset{\sim}{B} = \frac{1}{c}\,\underset{\sim}{n} \times \underset{\sim}{E} \;;\quad \underset{\sim}{E} = -\frac{c}{n^2}\,\underset{\sim}{n} \times \underset{\sim}{B}\,. \quad \text{— (17)}$$

In free space, the positive absolute magnitude of the $\hat{P}$ and $\hat{T}$ positive scalar, n, is unity. Using eqn. (17) the conjugate product is

$$\underset{\sim}{E} \times \underset{\sim}{E}^{*} = -\frac{c}{n^2}\,\underset{\sim}{E} \times \left(\underset{\sim}{n} \times \underset{\sim}{B}^{*}\right) = -\frac{\underset{\sim}{n}}{n}\,\frac{c\left(\underset{\sim}{E} \cdot \underset{\sim}{B}^{*}\right)}{n}\,. \quad \text{— (18)}$$

We note that the **vector** $\underset{\sim}{n}$ is a $\hat{P}$ negative, $\hat{T}$ negative, polar vector, defined as usual (7) as a propagation vector whose scalar magnitude is equal to the $\hat{P}$ negative scalar refractive index, n; and that the dot product $\underset{\sim}{E} \cdot \underset{\sim}{B}^{*}$ is a $\hat{T}$ and $\hat{P}$ negative pseudoscalar. The equation (18) reduces to

$$\underbrace{\left(\frac{E_0 B_0 c}{n}\right)_+}_{\text{scalar}} \underbrace{\underset{\sim}{e}_+}_{\substack{\text{axial}\\ \text{unit}\\ \text{vector}}} = \underbrace{\left(\frac{E_0 B_0 c}{n}\right)_-}_{\text{pseudoscalar}} \underbrace{\frac{\underset{\sim}{n}}{n}}_{\substack{\text{polar}\\ \text{unit}\\ \text{vector}}} \quad \text{— (19)}$$

in which we have designated the various symmetries. It follows algebraically that

$$\left(B_0\right)_+ \underset{\sim}{e}_+ = \left(B_0\right)_- \frac{\underset{\sim}{n}}{n} \quad \text{— (20)}$$

which can be rewritten in the notation of Section 2 as an example of **symmetry duality in the Maxwell equations:**

$$\left.\begin{aligned} \left(B_0\right)_+ \underset{\sim}{e}_+ &= \left(B_0\right)_- \underset{\sim}{e}_- \;; \\ \underset{\sim}{e}_- &\equiv \frac{\underset{\sim}{n}}{n}\,. \end{aligned}\right] \quad \text{— (21)}$$

This shows that classical vector $\underset{\sim}{B}_{\pi}$ can be defined simultaneously in terms of the unit axial vector $\underset{\sim}{e}_{+}$ and the unit polar vector $\underset{\sim}{e}_{-}$ which is related to the well known propagation vector $\underset{\sim}{\kappa}$, the photon linear momentum. In free space, with n = 1:

$$\underset{\sim}{B}_{\pi} = (B_0)_+ \underset{\sim}{e}_{+} = (B_0)_- \underset{\sim}{e}_{-} = (B_0)_- \frac{c}{\omega} \underset{\sim}{\kappa} \quad \text{— (22)}$$

demonstrating a duality between the classical angular and linear momentum of the plane wave. We shall see that this is none other than the classical equivalent of the well known de Broglie wave particle duality for the photon in the quantised field.

Before making the transition to the quantised field, however, another fundamentally new result emerges when we consider the well known definition [15] of the Stokes parameter $(S_3)_-$:

$$E_\alpha E_\beta^* - E_\beta E_\alpha^* = -i\, \epsilon_{\alpha\beta} (S_3)_- \quad \text{— (23)}$$

so that

$$(S_3)_- \equiv (E_0^2)_- \quad \text{— (24)}$$

is a **pseudoscalar** quantity implying inter alia the symmetry duality

$$\underset{\sim}{\Pi}^{(A)} = 2(E_0^2)_+ i \underset{\sim}{e}_{+} = 2(S_3)_- i \underset{\sim}{e}_{-} \quad \text{— (25)}$$

It follows directly that the magnetostatic vector $\underset{\sim}{B}_{\pi}$ can be defined in terms of $(S_3)_-$ as follows

$$\underset{\sim}{B}_{\pi} = \frac{(S_3)_-}{2E_0 c}\, \underset{\sim}{e}_- \equiv (B_0)_-\, \underset{\sim}{e}_- \quad -(26)$$

and we find that the role of B_0 as **pseudoscalar** is none other than the Stokes parameter S_3 scaled by an appropriate $\hat{P}$ and $\hat{T}$ positive scalar quantity. Thus $\underset{\sim}{B}_{\pi}$ can be defined in free space through the symmetry duality

$$\underset{\sim}{B}_{\pi} = (B_0)_+\, \underset{\sim}{e}_+ = \frac{(S_3)_-}{2E_0 c}\, \underset{\sim}{n} = \frac{(S_3)_-}{2E_0 c}\, \frac{c}{\omega}\, \underset{\sim}{\kappa} \quad -(27)$$

where the unit polar vector $\underset{\sim}{n}$ can be identified with the unit vector $\underset{\sim}{e}_-$ of this section.

We thus forge a novel and fundamental link between the pseudoscalar magnitude of $\underset{\sim}{B}_{\pi}$ and the pseudoscalar $(S_3)_-$.

4. SYMMETRY DUALITY AND WAVE PARTICLE DUALITY FOR THE PHOTON.

Eqn. (1) of the introduction shows that the photon's novel magnetic field operator, $\hat{B}_{\pi}$, is directly proportional to its well defined (6) angular momentum boson operator $\hat{J}$ through B_0 in its scalar representation, $(B_0)_+$, interpreted as the magnetic flux density amplitude of a single photon. The latter is a massless lepton which propagates at the speed of light and is not localised in space (16), unlike a massive lepton such as the electron or proton. These well known properties are contained in eqn. (1), in that B_0 varies with intensity I_0 for a beam of circularly polarised light containing one photon, and therefore B_0 for one photon depends on the beam cross section, a finite area. The eigenvalues of the operator $\hat{J}$ are well known to be $M_J\hbar$, $M_J = \pm 1$, there is no $M_J = 0$ component from relativistic considerations (6,7). Therefore the eigenvalues of $\hat{B}_{\pi}$ are $\pm(B_0)_+$; where $(B_0)_+$ is a scalar, the positive eigenvalue corresponds to one particular circular polarization, and vice versa (7), as in the well known convention for the operator $\hat{J}$.

We now use the well known result (6,7) that the eigenvalue of the linear momentum operator, $\hat{p}$, of the photon is:

l.c. $\underset{\sim}{p}$ bold

$$\underline{p} \equiv \langle \hat{p} \rangle = \hbar \underset{\sim}{\kappa} \qquad — (28)$$

where $\underset{\sim}{\kappa}$ is the wave vector as defined clasically in the preceding section. It follows straightforwardly from eqns. (22) and (28) that in free space ($n = 1$):

$$\hat{B}_{\pi} = (B_0)_+ \frac{\hat{J}}{\hbar} = \frac{(B_0)_-}{n} \frac{c}{\omega} \frac{\hat{p}}{\hbar} \quad — (29)$$

which expresses the duality of eqn. (22) in terms of quantum field theory, and shows that the $\hat{B}_{\pi}$ operator of the photon is simultaneously proportional to both its angular and linear momentum operators. Eqn. (29) summarises a duality in symmetry, linear / angular momentum, and wave / particle character. With the results

$$\left.\begin{array}{ll} \hat{T}\left((B_0)_+\right) = -\hat{T}\left((B_0)_-\right); \\ \hat{P}\left((B_0)_+\right) = -\hat{P}\left((B_0)_-\right); \end{array}\right\} \quad — (30)$$

eqn. (29) implies the free space relation

$$\hat{p} = n\frac{\omega}{c}\hat{J}; \; n = 1. \quad — (31)$$

The expectation value of $\hat{p}$ is therefore given by the expectation value of $\hat{J}$, which is $\pm\hbar$. Taking without loss of generality the positive eigenvalue $\hbar$, we have, with $n = 1$,

$$p = \frac{\omega}{c}\hbar \quad — (32)$$

which is the de Broglie wave particle duality for the photon.

We have therefore succeeded in relating directly the well known de Broglie wave-particle duality of quantum mechanics to the novel symmetry duality (22) of **classical** electromagnetic field theory. It has also been shown that the novel flux quantum

$\hat{B}_{\pi}$ is definable simultaneously in terms of $\hat{J}$ and $\hat{p}$, one operator being directly proportional to the other, implying that both must be quantised in the same way. In a sense therefore, $\hat{B}_{\pi}$ is the keystone of de Broglie's concept of duality for the photon.

Furthermore, contemporary elementary particle theory argues that the photon is a chiral entity, a massless lepton which travels in any frame of reference at c, and whose chirality, in consequence (17), is well defined in terms of the eigenvalues of Dirac's $\hat{\gamma}_5$ operator. The chirality of a lepton with mass (i.e. a "massive lepton") such as the electron is not well defined, leading to the idea (17) that mass itself is ill defined chirality. Well defined chirality in the photon can be thought of as being a consequence of superimposed linear and angular momentum, and eqn. (29) shows that there is a duality between these two fundamental quantities. It appears therefore that the novel $\hat{B}_{\pi}$ operator of the photon is a true chiral influence as defined by Barron (17), and is therefore fundamentally different in nature from a magnetostatic flux density, such as a magnetic field generated in an electromagnet. The latter is now known to be an example of a false chiral influence (17), and cannot, for example, be a cause of enantioselective synthesis. This is in contrast to the circularly polarised electromagnetic field, which le Bel in 1874 (18) conjectured to be a truly chiral influence, and which is now indeed well known to influence enantioselectivity in chemical reactions. The definition of the $\hat{B}_{\pi}$ operator in eqn. (29) also allows insight to the symmetry of natural optical activity, i.e.

circular dichroism and optical rotatory dispersion, as developed in the next section.

It may be conjectured, to end this section, that a magnetostatic flux quantum $\hat{B}_\pi$ is always carried by a massless lepton whose chirality can be precisely defined as the eigenvalues of the Dirac $\hat{\gamma}_5$ operator; and conversely that the massive lepton does not support $\hat{B}_\pi$ and does not have precisely defined eigenvalues of $\hat{\gamma}_5$. This conjecture would imply that fundamentally, $\hat{B}_\pi$ is always a consequence of the absence of mass. It would therefore follow that the neutrino (and anti-neutrino) carries a $\hat{B}_\pi$ field, but that the electron, neutron and proton do not. However, it is not clear whether the neutrino has a classical counterpart such as the classical electromagnetic plane wave, the counterpart of the photon, and if the parallel between photon and neutrino can be carried further, it would appear that the neutrino must also be thought of as unlocalised in space. This would imply inter alia that localisation in space implies the presence of mass and the absence of well defined chirality (or well defined eigenvalues of $\hat{\gamma}_5$), and that the absence of mass implies the absence of space localisation. Carrying the argument further, wave particle duality in a massive lepton such as the electron has been observed, because an electron beam can be diffracted, for example, but since the electron is localised and does not have well defined chirality, its wave nature must be fundamentally different from that of the photon, and in consequence, no $\hat{B}_\pi$ can be constructed or defined for the electron. Wave particle duality in the electron is therefore fundamentally different in nature from duality in the photon. The electron has a magnetic dipole moment as is well known, and which is proportional to the electron's spin angular momentum operator through the gyromagnetic ratio. We therefore conjecture that a massless lepton cannot support a magnetic dipole moment, because its effective gyromagnetic ratio would be infinite, but can support a magnetostatic flux quantum. The opposite is true for a massive lepton. With these assumptions, the $\hat{B}_{\pi}$ operator of a massless lepton would always be able to form an interaction hamiltonian operator to first order with the magnetic dipole moment operator of a massive lepton, an example being a photon beam interacting with an electron beam (5), or a neutrino beam with a neutron beam and so on, giving rise to measurable effects in principle.

The inference overall, therefore, is that a beam of massless leptons, for example photons or neutrinos, can magnetize but cannot be magnetized, whereas a beam of massive leptons cannot magnetize but can be magnetized.

The charge conjugation symmetry operator can be defined as $\hat{C}$ (which operates to reverse the sign of charge), and with this definition we recall the fundamental Luders Pauli Villiers Theorem (17)

$$\hat{C}\hat{P}\hat{T} = \hat{1}. \qquad (33)$$

As is well known, the violation of $\hat{P}$ has been observed (17) in a number of different ways, the violation of $\hat{T}$ in only one critical experiment (17). The violation of $\hat{P}$ leads to the result that the space inverted enantiomers of a truly chiral entity such as the photon or neutrino are not degenerate, or exactly the same in energy, because of the existence of the $\hat{P}$ violating electroweak force (17). In contrast, the space inverted "enantiomers" of a falsely or "pseudo" chiral entity, such as an ordinary magnetic field, are precisely the same in energy (17). Thus, it is important to note that the true enantiomer of the photon, or neutrino, is NOT generated by space inversion, or by application of the $\hat{P}$ operator, i.e. by reversing the linear momentum and keeping the angular momentum the same. Assuming that the photon is uncharged, so that $\hat{C}$ has no effect, its true or exact enantiomer must be generated by **simultaneous** $\hat{P}$ **and** $\hat{T}$ **violation** in order to conserve the validity of the Luders Pauli Villiers Theorem (33). The true enantiomer of the left handed photon is presumably, therefore, an object which must be designated the right handed "anti-photon", and there is a very small, but non-zero, energy difference between the left handed photon and the right handed photon. If the right handed photon is to be regarded as having a different energy from the left handed photon, then either: a) $\hat{P}$ has been violated and $\hat{T}$ and $\hat{C}$ have been conserved; or b) $\hat{T}$ has been violated and $\hat{P}$ and $\hat{C}$ have been conserved. Assuming that $\hat{C}$ has no effect on the photon, because it is uncharged, the combined operation $\hat{P}\hat{T}$ must be used to generate the right anti-photon from its true enantiomer, the left photon, and vice versa. The photon is an object whose chirality is generated only as a result of its simultaneous translational and rotational motion, and the novel $\hat{B}_{\pi}$ operator is a