363

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ELECTRODYNAMICS OF A ROTATING BODY: RELATIVISTIC THEORY OF CIRCULAR AND AXIAL BIREFRINGENCE

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The theory of the electrodynamics of a rotating body is used to show that there exists: 1) circular birefringence of purely relativistic origin, composed of dispersive aether drag and residual, ensemble averaged, magnetization; 2) non-relativistic circular birefringence due to the angular velocity of the body in the observer frame; 3) non-relativistic axial birefringence in chiral media due to the angular velocity of the body; 4) second order, relativistic equivalents of 2) and 3).

1. Introduction

It appears that J. J. Thompson was the first to analyse aether drag, when, a few years after the first Michelson Morley experiment, he considered[1] light passing through a medium that is rotating about an axis parallel to the propagation axis (Z) of the electromagnetic radiation. The angular drag per unit path length was obtained by Fermi[2] as

$$\xi = \frac{\Omega}{c} Z \left(n_1 - \frac{1}{n_0} \right) \equiv \frac{\Omega}{c} Z (n_g - n_\phi) \tag{1}$$

where n is the mean refractive index of the medium as it appears to an observer in the laboratory frame (X, Y, Z); Ω_Z is the angular frequency of the measuring radiation, and c the velocity of light. Player[3] later extended the analysis to dispersive aether drag, which causes a tiny circular birefringence of purely relativistic origin, measured carefully by R. V. Jones,[4] using a sensitive laser polarimeter. Player asserts[3] that there are two fundamental errors in Fermi's derivation, although he accepts that the final answer (1) obtained by Fermi appears to be correct, because it agrees with Thompson's.[1]

In this paper we use the well known classical theory of the electrodynamics of a rotating body to isolate several new effects which can be measured experimentally through circular and axial birefringence. In Sec. 2 a Maxwell equation is set

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up in the laboratory frame (X, Y, Z) of the stationary observer, using the usual Lorentz transformations from the frame of the rotating medium. Section 3 solves the equation for circular and axial birefringence effects of purely relativistic origin. Section 4 isolates new, purely relativistic, circular birefringence terms due to magnetization from the Lorentz transformation. These appear not to have been discussed by Player[3] or Fermi[2]; and what became the Lorentz transformation was probably not available to J. J. Thompson.[1] Section 5 introduces new non-relativistic circular and axial birefringence due to a rotating body, and Sec. 6 introduces new second order effects, the relativistic equivalent of (4).

2. The Lorentz Transforms and Maxwell Equations

Consider electromagnetic plane waves from a static source in (X, Y, Z) propagating through 1 metre of a cylinder of polarizable and magnetizable molecular material which is at rest. Assume initially that the magnetization and polarization are of similar order of magnitude, and that both must be considered in the initial theoretical development. Denote the laboratory frame by $K: (X, Y, Z, t)$ and let there be a frame $K': (X', Y', Z', t')$ which is moving at a velocity $\mathbf{v}(X, Y, Z, t)$. Assume that all fields in K do not have Z-directed components, and $v_Z = 0$. Furthermore, let the velocity $\mathbf{v} = v\hat{v}$, where $\hat{v}$ is a unit vector parallel to $\mathbf{v}$ and v is the magnitude of $\mathbf{v}$. Then, let

$$\beta = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} . \tag{2}$$

The Lorentz transforms are then defined as[5-10]

$$\begin{aligned}
\mathbf{E}' &= \hat{v}(\hat{v}\cdot\mathbf{E})(1-\beta) + \beta(\mathbf{E} + \mathbf{v}\times\mathbf{B}) ;\\
\mathbf{B}' &= \hat{v}(\hat{v}\cdot\mathbf{B})(1-\beta) + \beta(\mathbf{B} - \mathbf{v}\times\mathbf{E}/c^2) ;\\
\mathbf{D}' &= \hat{v}(\hat{v}\cdot\mathbf{D})(1-\beta) + \beta(\mathbf{D} + \mathbf{v}\times\mathbf{H}/c^2) ;\\
\mathbf{H}' &= \hat{v}(\hat{v}\cdot\mathbf{H})(1-\beta) + \beta(\mathbf{H} - \mathbf{v}\times\mathbf{D}) ;\\
\mathbf{P}' &= \hat{v}(\hat{v}\cdot\mathbf{P})(1-\beta) + \beta(\mathbf{P} - \mathbf{v}\times\mathbf{m}/c^2) ;\\
\mathbf{M}' &= \hat{v}(\hat{v}\cdot\mathbf{M})(1-\beta) + \beta(\mathbf{M} + \mathbf{v}\times\mathbf{P}) .
\end{aligned} \tag{3}$$

Here $\mathbf{E}$ denotes electric field strength in volts per meter; $\mathbf{B}$ magnetic flux density in tesla; $\mathbf{D}$ is electric displacement in coulombs per square metre; $\mathbf{H}$ is magnetic field strength in amperes per metre; $\mathbf{P}$ is electric polarization in coulombs per square metre; $\mathbf{M}$ is magnetization in amperes per metre. The velocity of light c is in metres per second, as is the frame velocity $\mathbf{v}$. In these S.I. units the equations of the Lorentz transform are self-consistent.[11]

In the approximation

$$v/c \ll 1 \tag{4}$$

they become

$$\begin{aligned}
\mathbf{E}' &= \beta(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \, ; \\
\mathbf{B}' &= \beta(\mathbf{B} - \mathbf{v} \times \mathbf{E}/c^2) \, ; \\
\mathbf{D}' &= \beta(\mathbf{D} + \mathbf{v} \times \mathbf{H}/c^2) \, ; \\
\mathbf{H}' &= \beta(\mathbf{H} + \mathbf{v} \times \mathbf{D}) \, ; \\
\mathbf{P}' &= \beta(\mathbf{P} - \mathbf{v} \times \mathbf{M}/c^2) \, ; \\
\mathbf{M}' &= \beta(\mathbf{M} + \mathbf{v} \times \mathbf{P}) \, .
\end{aligned} \tag{5}$$

Let Maxwell's equations in K' be

$$\begin{aligned}
\nabla' \times \mathbf{H}' &= \frac{\partial}{\partial t'} \mathbf{D}' \, , \\
\nabla' \times \mathbf{E}' &= -\frac{\partial}{\partial t'} \mathbf{B}', \ \mathbf{J}' = 0
\end{aligned} \tag{6}$$

and in K

$$\begin{aligned}
\nabla \times \mathbf{H} &= \frac{\partial}{\partial t} \mathbf{D} \, , \\
\nabla \times \mathbf{E} &= -\frac{\partial}{\partial t} \mathbf{B}, \\
\mathbf{J} &= 0
\end{aligned} \tag{7}$$

where $\mathbf{J}$ is the impressed current density in amperes per square metre; N the number of molecules per cubic metre in the sample; and μ_0 and ε_0 the permeability and permittivity in vacuo. In S.I. units these quantities are, respectively

$$\begin{aligned}
\mu_0 &= 4\pi \times 10^{-7} \quad \mathrm{Js^2C^{-2}m^{-1}} \\
\varepsilon_0 &= 8.854 \times 10^{-12} \quad \mathrm{J^{-1}C^2m^{-1}} \, .
\end{aligned} \tag{8}$$

They are related through

$$\varepsilon_0 \mu_0 = \frac{1}{c^2} \, . \tag{9}$$

In IUPAC convention, the electric field strength ($\mathbf{E}$) and magnetic flux density ($\mathbf{B}$) appearing in Eq. (1) satisfy the relations[12–14]

$$\begin{aligned}
&\mathbf{E}_{\mathrm{L}} = E_0(\mathbf{i} - i\,\mathbf{j})e^{i\phi_{\mathrm{L}}}; \ \mathbf{E}_{\mathrm{R}} = E_0(\mathbf{i} + i\,\mathbf{j})e^{i\psi_{\mathrm{R}}} \, , \\
&\mathbf{B}_{\mathrm{L}} = B_0(\mathbf{j} + i\,\mathbf{i})e^{i\phi_{\mathrm{L}}}; \ \mathbf{B}_{\mathrm{R}} = B_0(\mathbf{j} - i\,\mathbf{i})e^{i\phi_{\mathrm{R}}} \, , \\
&\phi_{\mathrm{R}} = \omega t - \mathbf{K}_{\mathrm{R}} \cdot \mathbf{r}; \ \phi_{\mathrm{L}} = \omega t - \mathbf{K}_{\mathrm{L}} \cdot \mathbf{r} \, .
\end{aligned} \tag{10}$$

Here ω is the angular frequency of the electromagnetic radiation in the static medium, $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in X and Y of the laboratory frame (X, Y, Z) of the static observer, t is the time in this frame, $\mathbf{K}_{\mathrm{R}}$ and $\mathbf{K}_{\mathrm{L}}$ are the wave vectors of the right (R) and left (L) circularly polarized components of the electromagnetic field, and $\mathbf{r}$ is a distance vector in the propagation axis (Z). Finally i denotes the root of minus one.

Now let the cylinder rotate at an angular velocity Ω_Z about the Z axis, while radiation propagates through it along the same Z axis. The velocity $\mathbf{v}$ in the Lorentz transforms (5) is defined by[3]

$$\mathbf{v} = \boldsymbol{\Omega} \times \mathbf{R}, \ \boldsymbol{\nabla} \cdot \mathbf{v} = 0; \ \boldsymbol{\nabla} \times \mathbf{v} = 2\boldsymbol{\Omega} \tag{11}$$

where R is the radius of the cylinder. Note that $\mathbf{v}$ has components in X and Y but not in Z:

$$v_X = -\Omega_Z Y, \ v_Y = \Omega_Z X \ . \tag{12}$$

Since $\mathbf{v}$ is clearly due to rotation in K it is a function of X and Y. In consequence, the rotating cylinder appears inhomogeneous as well as anisotropic to an observer in K, i.e. in the laboratory frame (X, Y, Z). To preserve the integrity of the material under Coriolis and centripetal acceleration therefore, gravitational and intermolecular forces should be considered additionally if $\mathbf{v}$ approaches c.[5–10] To all practical purposes, i.e. for attainable rod revolutions per minute, we have $|\mathbf{v}_{\max}| \ll c$ and the validity of the adiabatic Lorentz transformation is conserved.[7–10]

Consider a material in K' which has a polarization $\mathbf{P}' = \mathbf{D}' - \varepsilon_0\mathbf{E}'$ and a magnetization $\mathbf{M}' = \frac{1}{\mu_0}\mathbf{B}' - \mathbf{H}'$; then

$$\mathbf{P} = \beta(\mathbf{P}' + \varepsilon_0\mu_0\mathbf{v} \times \mathbf{M}') \tag{13}$$

and

$$\mathbf{M} = \beta(\mathbf{M}' - \mathbf{v} \times \mathbf{P}') \tag{14}$$

in K, showing that the effect of relativity is to introduce additional Lorentz magnetization and polarization, the terms $-\varepsilon_0\mu_0\mathbf{v} \times \mathbf{M}$ and $+\mathbf{v} \times \mathbf{P}$, respectively. The Maxwell curl equations in K' can be written as

$$\frac{1}{\mu_0}\boldsymbol{\nabla}' \times \mathbf{B}' - \boldsymbol{\nabla}' \times \mathbf{M}' = \varepsilon_0\frac{\partial}{\partial t'}\mathbf{E}' + \frac{\partial}{\partial t'}\mathbf{P}' \tag{15}$$

$$\boldsymbol{\nabla}' \times \mathbf{E}' = -\frac{\partial}{\partial t'}\mathbf{B}' \tag{16}$$

so

$$\frac{1}{\mu_0}\boldsymbol{\nabla}' \times (\mathbf{B} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{E}) - \boldsymbol{\nabla}' \times (\mathbf{M} + \mathbf{v} \times \mathbf{P}) = \varepsilon_0\frac{\partial}{\partial t'}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{\partial}{\partial t'}(\mathbf{P} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{M}) \tag{17}$$

and

$$\boldsymbol{\nabla}' \times (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\partial}{\partial t'}(\mathbf{B} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{E}) \ . \tag{18}$$

Note that **E**, **B**, **M**, and **P** belong to K, but the operators $\nabla' \times$ and $\frac{\partial}{\partial t'}$, belong to K'. However, if $|\mathbf{v}| \ll c$ we can remove the primes in Eqs. (17) and (18) to give

$$\frac{1}{\mu_0}\nabla \times (\mathbf{B} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{E}) - \nabla \times (\mathbf{M} + \mathbf{v} \times \mathbf{P}) = \varepsilon_0\frac{\partial}{\partial t}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{\partial}{\partial t}(\mathbf{P} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{M}) \tag{19}$$

and

$$\nabla \times (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\partial}{\partial t}(\mathbf{B} - \varepsilon_0\mu_0\mathbf{v} \times \mathbf{E}) \tag{20}$$

implying

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \frac{\partial}{\partial t}(\varepsilon_0\mathbf{E} + \mathbf{P}) + \left[\nabla \times (\mathbf{v} \times \varepsilon_0\mathbf{E}) + \nabla \times (\mathbf{M} + \mathbf{v} \times \mathbf{P}) + \varepsilon_0\frac{\partial}{\partial t}(\mathbf{v} \times \mathbf{B} - \mathbf{v} \times \mu_0\mathbf{M})\right] \tag{21}$$

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \left[\varepsilon_0\mu_0\frac{\partial}{\partial t}(\mathbf{v} \times \mathbf{E}) - \nabla \times (\mathbf{v} \times \mathbf{B})\right] \tag{22}$$

with ALL quantities now defined in (X, Y, Z). Note that Eq. (21) has four extra relativistic terms as a direct consequence of the Lorentz transformations. These are denoted within the braces in Eqs. (21) and (22). Two of these appear to have been discussed by Player,[3] but not the contribution coming from the initial assumption that the rod is polarizable, which introduces a purely relativistic magnetization term $\mathbf{v} \times \mathbf{P}$. The purely relativistic Lorentz polarization term $-\varepsilon_0\mu_0\mathbf{v} \times \mathbf{M}$, coming from the fact that the rod has been considered magnetizable, had also been previously left unconsidered. Additionally, the second Maxwell equation (22) has two purely relativistic terms denoted within the braces on the right-hand side. Player considered a quasimonochromatic optical disturbance in a isotropic medium of low absorption, where the displacement **D** was put directly proportional to the electric field strength **E**, with no explicit consideration of the (molecular) origin of **P**. We have considered the material equations (13) to (16) within the adiabatic Lorentz approximation, i.e. we have assumed that the material of the rod is both polarizable and magnetizable.

The quantities **E** and **B** appearing in Eqs. (19) to (22) continue to satisfy the plane wave equations (10), where **E** and **B** are mutually perpendicular quantities in (X, Y, Z). Note that Eqs. (1) and (19) to (22) are both written in (X, Y, Z), but if **E** and **B** are mutually perpendicular in one frame, they continue to be so[12] in any other frame of reference after Lorentz transformation.

The angular frequencies of the right and left circularly polarised components of the electromagnetic plane wave propagating in Z through a medium which is itself rotating about Z are affected through equal and opposite Doppler frequency shifts,[7] denoted $(\omega + \Omega_D)$ and $(\omega - \Omega_D)$ in frame (X, Y, Z) respectively for R and L components. The angular frequencies of the R and L components of the plane wave in the rotating medium appear different to the static observer.[3]

Finally in this section, we note that the Maxwell equation appears not to be written by Player[3] in S.I. units, whereas ours are written in the international (S.I.) convention adopted, for example, by Atkins.[11]

3. Purely Relativistic Circular Birefringence

We have been careful to define the quantities appearing in Eq. (21), which is now solved for circular birefringence of purely relativistic origin, the so-called aether drag.[3,4] The solution leads to new results based on the explicit consideration of P as in Eq. (13). It proves convenient to use the purely algebraic relations among vectors

$$\nabla \times (\mathbf{v} \times \mathbf{E}) = \mathbf{v}(\nabla \cdot \mathbf{E}) - \mathbf{E}(\nabla \cdot \mathbf{v}) + (\mathbf{E} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{E} \tag{23}$$

$$\nabla(\mathbf{E} \times \mathbf{v}) = \mathbf{E} \times (\nabla \cdot \mathbf{v}) + \mathbf{v} \times (\nabla \times \mathbf{E}) + (\mathbf{v} \cdot \nabla)\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{v} \tag{24}$$

which in the problem defined in Sec. 2 lead to

$$\nabla \times (\mathbf{v} \times \mathbf{E}) = \nabla(\mathbf{E} \cdot \mathbf{v}) - 2\mathbf{E} \times \Omega - \mathbf{v} \times (\nabla \times \mathbf{E}) \tag{25}$$

a result which appears to have been accepted[3] as the only origin of dispersive aether drag. (It will be shown later that the term $-(\mathbf{v} \times \mathbf{M})/c^2$ can also introduce aether drag in appropriate molecular ensembles.) Considering firstly the aether drag produced by (25), we write Eq. (21) as

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \frac{1}{\mu_0 c^2} (\nabla(\mathbf{E} \cdot \mathbf{v}) - 2\mathbf{E} \times \mathbf{\Omega}) + \ldots \tag{26}$$

and consider carefully the contributions of the two terms on the r.h.s. of (26), whose right and left components are

$$\frac{3}{c^2} \left(\Omega_Z E_X^R \mathbf{j} - \Omega_Z E_Y^R \mathbf{i}\right) + \mathbf{k} \text{ components}$$

and

$$\frac{3}{c^2} (\Omega_Z E_X^L \mathbf{j} - \Omega_Z E_Y^L \mathbf{i}) + \mathbf{k} \text{ components}$$

where c is the frame invariant velocity of light. The right and left wave vectors are

$$\mathbf{K}'_R = (\omega + \Omega_D)\frac{\mathbf{n}'_R}{c}, \ \mathbf{K}'_L = (\omega - \Omega_0)\frac{\mathbf{n}'_L}{c} \tag{27}$$

where $\mathbf{n}'_R$ and $\mathbf{n}'_L$ are the real refractive indices. The angular frequencies in (27) are different due to the opposite and equal Doppler effects.[3,12,13]

Appendix 1 considers carefully the contributions of the term $\nabla(\mathbf{E}\cdot\mathbf{v})$ in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, the unit vectors in X, Y, and Z. It is shown there that

$$\nabla(\mathbf{E}\cdot\mathbf{v}) = -\Omega_Z E_Y \mathbf{i} + \Omega_Z E_X \mathbf{j} + \left(\frac{\partial E_X}{\partial Z} v_X + \frac{\partial E_Y}{\partial Z} v_Y\right)\mathbf{k} . \tag{28}$$

The l.h.s. of (26) is therefore

$$iK'_{\mathrm{LZ}}(\mathbf{i} - i\mathbf{j})B_0 e^{i\phi_{\mathrm{L}}} \text{ or } iK'_{\mathrm{RZ}}(\mathbf{i} + i\mathbf{j})B_0 e^{i\phi_{\mathrm{R}}}$$

and comparing either $\mathbf{i}$ or $\mathbf{j}$ coefficients gives, self-consistently

$$(\omega - \Omega_0)n'_{\mathrm{LZ}} = 3\Omega_Z ; \; (\omega + \Omega_0)n'_{\mathrm{RZ}} = -3\Omega_Z . \tag{29}$$

The difference of these two equations gives the aether drag as the circular birefringence

$$(n'_{\mathrm{LZ}} - n'_{\mathrm{RZ}}) = 3\Omega_Z \left(\frac{1}{\omega - \Omega_{\mathrm{D}}} + \frac{1}{\omega + \Omega_{\mathrm{D}}}\right) . \tag{30}$$

Using the fundamental relation between E_0 and B_0 in S.I. units

$$E_0 = cB_0 , \tag{31}$$

finally gives the simple result

$$n'_{\mathrm{LZ}} = \frac{3\Omega_Z}{\omega - \Omega_{\mathrm{D}}}, \; n'_{\mathrm{RZ}} = \frac{-3\Omega_Z}{\omega + \Omega_{\mathrm{D}}} . \tag{32}$$

The angle of rotation of plane polarised radiation due to (30) is

$$\Theta = \frac{l\omega}{2c}(n'_{\mathrm{LZ}} - n'_{\mathrm{RZ}}) . \tag{33}$$

The results (30) and (32) appear similar to those obtained by Player (Eq. (1)) as follows

$$\xi \equiv \frac{\Theta}{l} = \frac{3}{2}\frac{\Omega_Z}{c}\left(\frac{\omega}{\omega - \Omega_{\mathrm{D}}} + \frac{\omega}{\omega + \Omega_{\mathrm{D}}}\right) . \tag{34}$$

In the approximation $\Omega_{\mathrm{D}} \ll \omega$, Eq. (34) reduces to

$$\xi = \frac{3\Omega_Z}{c} .$$

For visible radiation at 10^{15} radians per second; and for Ω_Z of about 100 radians per second the circular birefringence due to aether drag is of the order 10^{-8} from the simple result (34). The equivalent result obtained by Player[3] is recovered through the comparison of notation (see Eq. (1) ff.)

$$n_g - n_\phi \equiv \frac{3}{2}\left(\frac{\omega}{\omega - \Omega_{\mathrm{D}}} + \frac{\omega}{\omega + \Omega_{\mathrm{D}}}\right) \tag{35}$$

which links the notation used by Player and ours. With this equivalence the results are identical with those of Player for the angular aether drag per unit path length. Equation (35) is developed further in Appendix 2.

4. Relativistic Circular Birefringence Due to Magnetization

The origin of this new contribution is the term $-\mu_0\epsilon_0\mathbf{v}\times\mathbf{M}$ of the right-hand side of Eq. (21). This is magnetization purely due to the Lorentz transform of fundamental relativity theory.[12] The magnetization $\mathbf{M}$ is multiplied by $\mathbf{v}$ and the product may be significant in magnitude in frame (X, Y, Z) of the static observer. Following concepts described elsewhere,[15,16] $\mathbf{M}$ is assumed to be molecular in origin and expanded in a double Taylor series in $\mathbf{E}$ and $\mathbf{B}$. To first order, in tensor notation,

$$M_i = a_{ij}B_j + b_{ij}E_j + \ldots \tag{36}$$

where a_{ij} and b_{ij} are respectively even and odd parity molecular property tensor,[17–20] defined in (X, Y, Z). In general both linear terms of Eq. (36) contribute to aether drag, together with higher order terms not considered here. The effect is illustrated with the $\mathbf{i}$ and $\mathbf{j}$ terms of

$$\mathbf{v}\times\mathbf{M} = v_Y M_Z\,\mathbf{i} - v_X M_Z\,\mathbf{j} \quad (M_i = a_{ij}B_j + \ldots) \tag{37}$$

which depend on off-diagonal components of the even parity molecular property tensor a_{ij}. These survive isotropic ensemble averaging[19,20] in general in certain molecular crystalline point groups. The effect could therefore be observed in a rotating achiral single crystal of appropriate structure. In a chiral single crystal there are also aether drag contributions from off-diagonal components of b_{ij}. Comparing i and j coefficients of the terms $\frac{1}{\mu_0}\nabla\times\mathbf{B}$ and $-\frac{\partial}{\partial t}(\mathbf{v}\times\mathbf{M})/c^2$ respectively on the left- and right-hand sides of Eq. (21) gives

$$\langle n'_{\mathrm{L}} - n'_{\mathrm{R}}\rangle = 2\mu_0\Omega_Z Y N\langle b_{ZY}\rangle \tag{38}$$

for the circular birefringence, a result which appears not to have been described in the literature. We refer to this as aether drag due to Lorentz magnetization, or LM aether drag.

5. Non-Relativistic Circular and Axial Birefringence Due to Angular Velocity

In direct analogy with the well known Faraday effect,[17,21] which is circular birefringence due to a static magnetic field, there is non-relativistic circular birefringence due to the angular velocity Ω_Z which appears through non-relativistic polarisation and magnetization in the Maxwell equation. Additionally, there is axial birefringence of the type first described by Wagniere and Meier[22–24] for a static magnetic field applied parallel and anti-parallel with unpolarised probe electromagnetic radiation propagating in Z. The angular velocity vector Ω_Z has the same parity (P)

and reversality (T) symmetry[25] as the static magnetic field, and therefore circular and axial birefringence due to Ω_Z conserves T in all ensembles.[17] Circular birefringence due to Ω_Z conserves P in all ensembles, and axial birefringence due to Ω_Z in chiral ensembles.

Under appropriate conditions, therefore, aether drag as discussed by Player[3] and observed by Jones[4] in a rotating glass rob is accompanied both by relativistic LM aether drag and these non-relativistic effects caused by polarisation and magnetization.

The theory of the Faraday effect is well developed[17] and can be adapted straightforwardly for circular birefringence due to Ω_Z through the introduction of new fundamental molecular property tensors. The same technique can also be used for non-relativistic axial birefringence due to Ω_Z. It is therefore assumed[17,21] that the molecular polarisability tensor α_{1ij}, molecular Roesenfeld tensor α_{2ij}, and electric dipole electric quadrupole tensor A_{ijk} are perturbed by Ω_Z as follows

$$\alpha_{1ij}(\Omega_Z) = \alpha_{1ij} + \alpha_{1ijZ}\Omega_Z + \ldots \tag{39}$$

$$\alpha_{2ij}(\Omega_Z) = \alpha_{2ij} + \alpha_{2ijZ}\Omega_Z + \ldots \tag{40}$$

$$A_{ijk}(\Omega_Z) = A_{ijk} + A_{ijkZ}\Omega_Z + \ldots \tag{41}$$

to first order in Ω_Z. The latter's effect is therefore mediated by new higher rank molecular property tensors which are caused by circular and axial birefringence. The theory also accounts for the magnetization due to E_j

$$M_i = \alpha^*_{2ij}E_j + \ldots \tag{42}$$

and the induction of a molecular electric quadrupole moment

$$\Theta_{ij} = A_{kij}E_j + \ldots \tag{43}$$

both mediated by the new higher rank tensors in Eqs. (39) to (41). With these definitions it is straightforward to adapt the existing theory of the Faraday effect[17] and Wagnière-Meier effect[22–25] to give the following ensemble averaged expressions for non-relativistic circular and axial birefringence due to Ω_Z.

Ensemble averaged non-relativistic circular birefringence due to the angular velocity Ω_Z is present in all ensembles and is mediated by even-parity scalar elements of the new molecular property tensors as follows

$$\begin{aligned}&\langle n'_{\mathrm{L}} - n'_{\mathrm{R}}\rangle_{\uparrow\uparrow} - \langle n'_{\mathrm{L}} - n'_{\mathrm{R}}\rangle_{\uparrow\downarrow}\\ &= 2\mu_0 c^2 N\Omega_Z \left(\langle\alpha''_{1XYZ}\rangle - \frac{\omega}{2c}\left(\langle A'_{XYZZ}\rangle - \langle A_{YXZZ}\rangle\right)\right) .\end{aligned} \tag{44}$$

Its axial counterpart is measured by unpolarised probe radiation parallel and antiparallel with Ω_Z and is mediated by ensemble averaged scalar components of odd-parity molecular property tensors which exist in chiral ensembles only:

$$\begin{aligned}\langle n_{\downarrow\downarrow} - n_{\uparrow\downarrow}\rangle = \mu_0 c N\Omega_Z \Big[&\langle\alpha'_{2XYZ}\rangle - \langle\alpha'_{2YXZ}\rangle + \frac{\omega}{6}\left(\langle A''_{XXZZ} + \langle A''_{ZXXZ}\rangle\right.\\ &\left. + \langle A''_{YYZZ}\rangle + \langle A''_{ZYYZ}\rangle\right)\Big] .\end{aligned} \tag{45}$$

6. Second Order Relativistic Effects

These are generated by considering perturbations such as

$$a_{2ij}(\Omega_Z) = a_{2ij} + a_{2ijZ}\Omega_Z \tag{46}$$

in Eq. (25) in frame (X, Y, Z) of the static observer, producing immediately results such as

$$\begin{aligned}\langle n'_{\mathrm{L}} - n'_{\mathrm{R}}\rangle_{\mathrm{L}M\uparrow\uparrow} - \langle n'_{\mathrm{L}} - n'_{\mathrm{R}}\rangle_{\mathrm{L}M\uparrow\downarrow} \\ = 4\mu_0\Omega_Z^2 Y N\langle a_{2ZYZ}\rangle + \dots\end{aligned} \tag{47}$$

proportional to Ω_Z^2 and to a three rank ensemble average. The latter may survive ensemble averaging in certain crystal symmetries, but not in an isotropic liquid spinning with net Ω_Z. The contribution (5) to circular birefringence is a second order effect, therefore, produced by a combination of relativistic and non-relativistic effects.

7. Discussion

The interesting experimental work of Jones,[4] and theoretical work of Player[3] appears to form a solid base upon which to explore the several contributions to birefringence discussed in this paper. Specifically, if the rotating rod of low absorbing material investigated by Jones were to be replaced by an appropriate chiral or achiral molecular material, several new effects could be isolated, some relativistic, and some of molecular origin, providing information, therefore, on fundamental molecular properties. Apart from the aether drag isolated by Player, which we have derived as Eq. (35), there is circular birefringence due to Lorentz magnetization, Eq. (38), which is relativistic, but depends on the molecular property tensor scalar component $\langle b_{ZY}\rangle$, which survives ensemble averaging in certain crystal symmetries. Then there is the non-relativistic circular birefringence (44), which exists in chiral and achiral liquids, spinning in a container, and the axial birefringence (45), which exists in chiral liquids. Finally there are second order effects exemplified by (47).

It is probable that the apparatus built by Jones[4] is sensitive enough to characterise these new effects and to produce their spectra as a function of frequency of probe radiation. For example the angular drag per unit length from Eq. (38) is given by

$$\frac{\Theta}{l} = \mu_0\frac{\Omega_Z}{c}\left(\omega Y\langle b_{ZY}\rangle N\right) \tag{48}$$

which is proportional to the Y dimension of the rotating rod, and to the ensemble average $\langle b_{ZY}\rangle$, a material property, multiplied by the number of molecules per cubic metre, N. This form is quite different from that in Eq. (1), showing that the Lorentz magnetization cannot be described as a simple modification of the index of refraction.

There will be similar considerations, which we have not analysed here, for the Lorentz polarization caused by the assumption that the rod is magnetizable in general, as well as polarizable. The analysis leading to Eq. (38) has assumed implicitly that the magnitude of the magnetization is small compared with that of the polarization, and a material can always be chosen to fit this restriction. If this assumption is not made, the relativistic Maxwell equations become intractable without recourse to numerical solution. It is hoped that this will be the subject of a separate paper within the framework of general relativity.

A computer simulation of these effects would provide estimates of the frequency dependent polarization effects,[26] as in recent work by Evans and Wagnière.[15,26] We expect similar frequency dependent effects due to relativistic Lorentz magnetization and polarization. This is clear from the nature of the molecular property tensor ensemble average $\langle b_{ZY} \rangle$ in Eq. (48), which in semiclassical theory is a dynamical (frequency dependent) quantity.[17] We note from Eq. (48) that the Lorentz magnetization effect appears to be of the same order of magnitude, at least, as the effect of Eq. (1), depending on the order of magnitude of the molecular property $\langle b_{ZY} \rangle$.

Finally, an order of magnitude estimate can be made from Eq. (48) if we take an order of magnitude for $\langle b_{ZY} \rangle$ of about $10^{-34} A^2 J^{-1} m^3 s$,[25] N about 6×10^{26} molecules per cubic metre, a visible frequency ω of about 10^{15} radians per second, a rotational frequency Ω_Z of about 100 revolutions per second, and Y about 0.01 m (one cm).

These give a value of about 10^{-7} radians per metre for the angular aether drag per unit length of rod due to the Lorentz magnetization term alone. This is about the same order of magnitude as the Fermi result (Eq. (1)) and very much depends on the order of magnitude taken for $\langle b_{ZY} \rangle$. It has also been assumed that the value of Ω_Z is only 100 revolutions per second, or 6000 revs. per minute (r.p.m.), and that the radius of the rod, Y, is only about a centimetre. It is quite possible therefore, to observe this effect, because it is within the limits of the apparatus built by R. V. Jones.[4]

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Appendix 1

The term $\nabla(\mathbf{E} \cdot \mathbf{v})$ can be evaluated straightforwardly, but for clarity, the algebra is summarised as follows. We have

$$\nabla(\mathbf{E} \cdot \mathbf{v}) = \frac{\partial}{\partial x}(E_x v_x + E_y v_y)\mathbf{i} + \frac{\partial}{\partial y}(E_x v_x + E_y v_y)\mathbf{j} + \frac{\partial}{\partial z}(E_x v_x + E_y v_y)\mathbf{k} \quad \text{(A1)}$$

with

$$\frac{\partial}{\partial x}(E_x v_x) = \frac{\partial E_x}{\partial x} v_x + E_x \frac{\partial v_x}{\partial x} \quad \text{(A2)}$$

and so on. The electric field components are

$$E_x^+ = E_0 e^{i\phi_+}; \quad E_x^- = E_0 e^{i\phi_-}$$
$$E_y^+ = -E_0 i e^{i\phi_+}; \quad E_y^- = E_0 i e^{i\phi_-} \tag{A3}$$

so that

$$\frac{\partial E_x^\pm}{\partial x} = \frac{\partial E_x^\pm}{\partial y} = \frac{\partial E_y^\pm}{\partial x} = \frac{\partial E_y^\pm}{\partial y} = 0 \tag{A4}$$

but

$$\frac{\partial E_x^\pm}{\partial z} \neq \frac{\partial E_y^\pm}{\partial z} \neq 0\,. \tag{A5}$$

Further results

$$\frac{\partial v_x}{\partial y} = -\frac{\partial v_y}{\partial x} = -\Omega_z; \ \frac{\partial v_x}{\partial x} = \frac{\partial v_x}{\partial z} = \frac{\partial v_y}{\partial y} = \frac{\partial v_y}{\partial z} = \frac{\partial v_Z}{\partial x} = \frac{\partial v_Z}{\partial y} = \frac{\partial v_Z}{\partial z} = 0 \tag{A6}$$

finally provide

$$\nabla(\mathbf{E}\cdot\mathbf{v}) = -\Omega_z E_y \mathbf{i} + \Omega_z E_x \mathbf{j} + \left(\frac{\partial E_x}{\partial z} v_x + \frac{\partial E_y}{\partial z} v_y\right)\mathbf{k} \tag{A7}$$

which is Eq. (28) of the text.

Appendix 2

The comparison of notation, Eq. (35) of the text, is developed using the relativistic theory of the Doppler effect[13] to express the Doppler angular frequency as

$$\Omega_D = \frac{\Omega_Z|\mathbf{R}|\omega}{2c} \tag{B1}$$

where Ω_Z is the frequency of rotation of the rod in revolutions per second and $\mathbf{R}$ its radius in metres. The quantity

$$|\mathbf{v}_{\text{rod}}| = \Omega_Z|\mathbf{R}| \tag{B2}$$

is a linear velocity, and in consequence, the ratio

$$n_2 = \frac{2c}{|\mathbf{v}_{\text{rod}}|} \tag{B3}$$

is a relativistically generated refractive index, related directly to, and generated by, the angular velocity of the rod. Equation (35) becomes

$$n_g - n_\phi \equiv \frac{3}{2}\left(\frac{n_2}{1+n_2} - \frac{n_2}{1-n_2}\right) \tag{B4}$$

so that in our notation

$$n_g \equiv \frac{3}{2}\frac{n_2}{1+n_2}; \quad n_\phi \equiv \frac{3}{2}\frac{n_2}{1-n_2} \tag{B5}$$

thus linking our notation to that of Player and Fermi in terms of a refractive index n_2, which is generated by the Doppler frequency shift due to the rotating rod. Note that if the rod is not rotating, Ω_D vanishes, and there is no relativistic refractive index difference. The difference due Ω_Z is purely relativistic, and must be distinguished from other sources of birefringence, such as dispersion due to $dn/d\lambda \neq 0$, which are not relativistic. The non-relativistic birefringence remains measurable in a rod which is not rotating with respect to the observer and which does not generate a Doppler frequency shift Ω_D.

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