ECE Engineering Model - Linearized Equations

The Basis for Electromagnetic and Mechanical Applications

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ECE Field Equations – Vector Form

$$\nabla \cdot \mathbf{B} = \mu_0 \rho_{eh} = \rho_{eh}'$$ Gauss Law

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \mathbf{j}_{eh} = \mathbf{j}_{eh}'$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}_e$$ Ampère - Maxwell Law

„Material“ Equations

$$\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E}$$ Dielectric Displacement

$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$ Magnetic Induction

Field-Potential Relations

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \Phi - \omega_0 \mathbf{A} + \boldsymbol{\omega} \Phi$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}$$

Potentials and Spin Connections

$\mathbf{A}$: Vector potential

Φ: scalar potential

$\boldsymbol{\omega}$: Vector spin connection

ω_0: Scalar spin connection

ECE Field Equations in Terms of Potential

Gauss Law:

$$\nabla\cdot(\boldsymbol{\omega}\times\mathbf{A})=0$$

Faraday Law of Induction:

$$-\nabla\times(\omega_0\mathbf{A})+\nabla\times(\boldsymbol{\omega}\Phi)-\frac{\partial\boldsymbol{\omega}}{\partial t}\times\mathbf{A}-\boldsymbol{\omega}\times\frac{\partial\mathbf{A}}{\partial t}=0$$

Coulomb Law:

$$-\nabla\cdot\frac{\partial\mathbf{A}}{\partial t}-\nabla\cdot(\omega_0\mathbf{A})-\Delta\Phi+\nabla\cdot(\boldsymbol{\omega}\Phi)=\frac{\rho_e}{\varepsilon_0}$$

Ampère-Maxwell Law:

$$\nabla\times\nabla\times\mathbf{A}-\nabla\times(\boldsymbol{\omega}\times\mathbf{A})$$

$$+\frac{1}{c^2}\left(\frac{\partial^2\mathbf{A}}{\partial t^2}+\frac{\partial(\omega_0\mathbf{A})}{\partial t}+\nabla\frac{\partial\Phi}{\partial t}-\frac{\partial(\boldsymbol{\omega}\Phi)}{\partial t}\right)=\mu_0\mathbf{J}_e$$

Variable Transformation

Define new functions:

$$\mathbf{F} := \boldsymbol{\omega}\Phi$$

$$\mathbf{G} := \boldsymbol{\omega} \times \mathbf{A}$$

$$\mathbf{K} := \omega_0 \mathbf{A}$$

Old function set (8 comp.):

$\mathbf{A}, \Phi, \boldsymbol{\omega}, \omega_0$

New function set (13 comp.):

$\mathbf{A}, \Phi, \mathbf{F}, \mathbf{G}, \mathbf{K}$

➔Linear field-potential relations:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi - \mathbf{K} + \mathbf{F}$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \mathbf{G}$$

Transformed ECE Field Equations

Gauss Law :

$$\nabla \cdot \mathbf{G} = 0$$

Faraday Law of Induction :

$$-\nabla \times \mathbf{K} + \nabla \times \mathbf{F} - \frac{\partial \mathbf{G}}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \nabla \cdot \mathbf{K} - \Delta \Phi + \nabla \cdot \mathbf{F} = \frac{\rho_e}{\varepsilon_0}$$

Ampère – Maxwell Law :

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{G} + \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{\partial \mathbf{K}}{\partial t} + \nabla \frac{\partial \Phi}{\partial t} - \frac{\partial \mathbf{F}}{\partial t}\right) = \mu_0 \mathbf{J}_e$$

Physical Units

$$[\mathbf{E}] = \frac{V}{m}$$

$$[\mathbf{B}] = T = \frac{V \cdot s}{m^2} = \frac{N}{A \cdot m}$$

$$[\mathbf{D}] = \frac{C}{m^2}, \quad [\mathbf{H}] = \frac{A}{m}$$

$$[\Phi] = V$$

$$[\mathbf{A}] = \frac{Vs}{m}$$

$$[\omega_0] = \frac{1}{s}$$

$$[\boldsymbol{\omega}] = \frac{1}{m}$$

$$[\mathbf{F}] = \frac{V}{m} = [\mathbf{E}]$$ „spin-electric Field“

$$[\mathbf{G}] = \frac{V \cdot s}{m^2} = [\mathbf{B}]$$ „spin-magnetic Field“

$$[\mathbf{K}] = \frac{V}{m} = [\mathbf{E}]$$ „magneto-electric Field“

ECE Field Equations – Static Case

8 equations
13 functions

Gauss Law:

$$\nabla \cdot \mathbf{G} = 0$$

Faraday Law of Induction:

$$-\nabla \times \mathbf{K} + \nabla \times \mathbf{F} = 0$$

Coulomb Law:

$$-\nabla \cdot \mathbf{K} - \Delta\Phi + \nabla \cdot \mathbf{F} = \frac{\rho_e}{\varepsilon_0}$$

Ampère-Maxwell Law:

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{G} = \mu_0 \mathbf{J}_e$$

ECE Field Equations – Pure Electric Case

Gauss Law :

– – –

Faraday Law of Induction :

$$\nabla \times \mathbf{F} = 0$$

Coulomb Law :

$$-\Delta\Phi + \nabla \cdot \mathbf{F} = \frac{\rho_e}{\varepsilon_0}$$

Ampère - Maxwell Law :

$$\frac{1}{c^2}\left(\nabla \frac{\partial \Phi}{\partial t} - \frac{\partial \mathbf{F}}{\partial t}\right) = \mu_0 \mathbf{J}_e$$

Static:
4 equations
4 functions

Dynamic:
4 equations
4 functions

ECE Field Equations – Pure Magnetic Case

8 equations
9 functions

Gauss Law:

$$\nabla \cdot \mathbf{G} = 0$$

Faraday Law of Induction:

$$-\nabla \times \mathbf{K} - \frac{\partial \mathbf{G}}{\partial t} = 0$$

Coulomb Law:

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \nabla \cdot \mathbf{K} = \frac{\rho_e}{\varepsilon_0}$$

Ampère-Maxwell Law:

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{G} + \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{\partial \mathbf{K}}{\partial t}\right) = \mu_0 \mathbf{J}_e$$

ECE Field Equations –Magnetostatic Case

G is vortex field like **B**, **K** is source field like **E**

4 equations
6 functions

Gauss Law:

$$\nabla \cdot \mathbf{G} = 0$$

Faraday Law of Induction:

$$\nabla \times \mathbf{K} = 0$$

Coulomb Law:

$$-\nabla \cdot \mathbf{K} = \frac{\rho_e}{\varepsilon_0}$$

Ampère - Maxwell Law:

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{G} = \mu_0 \mathbf{J}_e$$

4 equations
3 functions,
analogous
to electro-
static E field

ECE Field Equations –Magnetostatic Case with Conductivity Term

$$\mathbf{J}_e = \sigma\mathbf{E} = \sigma\left(-\frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A}\right) = \sigma\left(-\frac{\partial \mathbf{A}}{\partial t} - \mathbf{K}\right)$$

8 equations
9 functions

Gauss Law:

$$\nabla \cdot \mathbf{G} = 0$$

Faraday Law of Induction:

$$\nabla \times \mathbf{K} = 0$$

Coulomb Law:

$$-\nabla \cdot \mathbf{K} = \frac{\rho_e}{\varepsilon_0}$$

Ampère - Maxwell Law:

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{G} = -\mu_0 \sigma \mathbf{K}$$